AP Calculus

Chapter 4

Section 4-1

May 13-10:02 PM

Derivatives of Compositions

Find $\frac{dy}{dx}$. $y = (5x)^3$

$y = 125x^3$

$\frac{dy}{dx} = 375x^2$

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Derivatives of Compositions

Find $\frac{dy}{dx}$. $y = (4x^3+7\ )^2 =$

$y=(4x^3)^2+56x^3\ +49$

$y= 16x^6+56x^3+49$

$y'=96x^5+168x^2$

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How do variables x and u relate?

How do the three derivatives relate?

$y = 15x - 6$ $y = 3u$

$\frac{15x-6}{3}=\frac{3u}{3}$ $\frac{dy}{dx}=15$ $\frac{dy}{du}=3$

$5x-2=u$

$5=\frac{du}{dx}$

$15=3\cdot 5$

$\frac{dy}{dx}=\frac{dy}{\cancel{du}}\cdot\frac{\cancel{du}}{dx}$

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Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

OR

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

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Chain Rule

Find $\frac{dy}{dx}$. $y = (5x)^3 \rightarrow y = u^3$

$u = 5x$ $\frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = 5$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{dy}{dx} = 3u^2 \cdot 5$

$3(5x)^2 \cdot 5$

$3 \cdot 25x^2 \cdot 5 = 375x^2$

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$y=(5x-1)^{13}$

$y=u^{13}$

$u=5x-1$

$\frac{dy}{du}=13u^{12}$

$\frac{du}{dx}=5$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=13u^{12}\cdot 5$

$=13(5x-1)^{12}\cdot 5$

$=65(5x-1)^{12}$

$13(5x-1)^{12}\cdot 5$

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Chain Rule

Find $\frac{dy}{dx}$. $y = (4x^3+7)^2$

$u=4x^3+7$

$y=u^2$

$\frac{du}{dx}=12x^2$

$\frac{dy}{du}=2u$

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=2u(12x^2)$

$\frac{dy}{dx}=2(4x^3+7)(12x^2)$

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Chain Rule

Find $\frac{dy}{dx}$. $y = \cos^5(x)$ $= [\cos(x)]^5$

$y = u^5$

$\frac{dy}{du} = 5u^4$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4(-\sin x)$

$= 5\cos^4(x)(-\sin x)$

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

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Degrees vs. Radians

Find $\frac{dy}{dx}$. $y = \sin(x^o)$ **Hint: x^o = ?? radians

$y = \sin\left(\frac{x \cdot \pi}{180}\right)$

$u = \frac{x\pi}{180} = x \cdot \frac{\pi}{180}$

$y = \sin(u)$ $\frac{du}{dx} \leftarrow u' = \frac{\pi}{180}$

$\frac{dy}{du} \leftarrow y' = \cos(u)$

$\frac{dy}{dx} = \cos(u) \cdot \frac{\pi}{180}$

$\frac{dy}{dx} = \cos\left(\frac{x\pi}{180}\right) \cdot \frac{\pi}{180}$

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Chain Rule

Find $\frac{dy}{dx}$. $y = \tan^5(x+x^{-2})$

$y = \left[\tan(x+x^{-2})\right]^5$

$y = u^5$ $u = \tan(x+x^{-2})$ $v = x+x^{-2}$

$y' = 5u^4$ $u = \tan(v)$ $v' = 1-2x^{-3}$

$u' = \sec^2(v)$

$\frac{dy}{dx} = 5u^4\sec^2(v)(1-2x^{-3})$

$= 5(\tan(x+x^{-2}))^4\sec^2(x+x^{-2})(1-2x^{-3})$

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Chain Rule

Find $\frac{dy}{dx}$. $y = [x^2 - 2\cot(x^2+1)]^3$

$y = u^3$ $u = x^2 - 2\cot(x^2+1)$ $v = x^2+1$

$y' = 3u^2$ $v' = 2x$

$u = v-1-2\cot(v)$ $v-1 = x^2$

$u' = 1-2(-\csc^2(v))$

$u' = 1+2\csc^2(v)$

$\frac{dy}{dx} = 3u^2(1+2\csc^2(v))(2x)$

$= 3(x^2-2\cot(x^2+1))(1+2\csc^2(x^2+1))(2x)$

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Chain Rule and Quotient Rule

Find $\frac{dy}{dx}$. $y = \frac{\sec(x^2 - 25)}{x^3 + 3x^2 + 1}$

$u = \sec(x^2 - 25)$

$t = x^2 - 25$

$v = x^3 + 3x^2 + 1$

$v' = 3x^2 + 6x$

$u = \sec(t)$

$\frac{du}{dt} = \sec(t)\tan(t)$ $t' = 2x$

$u' = \frac{du}{dx} = \sec(x^2-25)\tan(x^2-25)\,2x$

$\frac{vu' - uv'}{v^2}$

$= \frac{(x^3+3x^2+1)(\sec(x^2-25)\tan(x^2-25)2x) - \sec(x^2-25)(3x^2+6x)}{(x^3+3x^2+1)^2}$

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Find the equation for the tangent line at $x = 5$.

Support your answer graphically.

$y = 3x^2 - \cos\left(\frac{1}{10}\pi \cdot x\right) - 7x$

$y' = 6x + \sin\left(\frac{1}{10}\pi x\right) \cdot \frac{1}{10}\pi - 7$

$y' = 30 + \sin\left(\frac{\pi}{2}\right) \cdot \frac{1}{10}\pi - 7$

slope $\longrightarrow y' = 23 + \frac{1}{10}\pi$

point $y = 3(5)^2 - \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} - 7(5)$

(5, 40) $y = 40$

$y - 40 = \left(23 + \frac{1}{10}\pi\right)(x - 5)$

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Derivatives of Parametric Curves

If y and x are both defined in terms of t, all three derivatives exist, and dx/dt ≠ 0,

then

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \qquad y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Oct 24-8:18 AM

Parametric Equations

Find $\frac{dy}{dx}$. $x = \cos(t)$, $y = \sec(t)$ (Simplify your answer)

$x' = -\sin(t)$ $\quad y' = \sec(t)\tan(t)$

$$\frac{dy}{dx} = \frac{dy/\cancel{dt}}{dx/\cancel{dt}} = \frac{\sec(t)\tan(t)}{-\sin(t)} = \frac{1}{-\cancel{\sin t}} \cdot \frac{1}{\cos t} \cdot \frac{\cancel{\sin t}}{\cos t}$$

$$\frac{dy}{dx} = -\frac{1}{\cos^2 t} = -\sec^2(t)$$

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Parametric Equations

Find $\frac{dy}{dx}$ at $t = \pi$. $x = 3t - \cos(t)$, $y = 3t\cos(t)$

$x' = 3 + \sin t$ $y' = 3t(-\sin t) + 3\cos t$

$$\frac{dy}{dx} = \frac{-3t\sin(t) + 3\cos t}{3 + \sin(t)}$$

$$\frac{\cancel{-3\pi(0)} + 3(-1)}{3} = -1$$

der slope of tan

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RULE 9 Power Rule for Rational Powers of *x*

If n is any rational number, then

$$\frac{d}{dx}x^n = \boxed{nx^{n-1}.}$$

If $n < 1$, then the derivative does not exist at $x = 0$.

***The proof of this rule will appear in section 4-2

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Rational Powers

Find $\frac{dy}{dx}$. $y = \sqrt{x} = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

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Rational Powers

Find $\frac{dy}{dx}$. $y = \sqrt[5]{x^9} = x^{9/5}$

$y' = \frac{9}{5}x^{4/5}$

$= \frac{9}{5}\sqrt[5]{x^4}$

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Rational Powers

Find $\frac{dy}{dx}$. $y = x^{7/5}$

$$y' = \frac{7}{5}x^{2/5}$$

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Rational Powers and the Chain Rule

Find $\frac{dy}{dx}$. $y = x\sqrt[3]{x - 3}$

$$y = x(x-3)^{1/3}$$

$$y' = \underbrace{(x)}_{1^{st}}\underbrace{\tfrac{1}{3}(x-3)^{-2/3}(1)}_{\text{der. of 2nd}} + \underbrace{(1)}_{\text{der. 1st}}\underbrace{(x-3)^{1/3}}_{\text{2nd}}$$

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Homework

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1-11 odd, 33-47 odd, 50-56, 58, 59, 70-74

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