AP Calculus

Chapter 4

Section 4-2

May 13-10:02 PM

Differentiating a function of y with respect to x.

Find $\frac{dz}{dx}$. $z = y^3$, $\frac{dy}{dx} = 11x$.

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

$$\frac{dz}{dx} = 3y^2 \cdot 11x$$

(10, 4)

$3 \cdot 4^2 \cdot 11 \cdot 10$

$48 \cdot 110 = 5280$

Oct 6-9:10 PM

Implicit Differentiation

We can generalise this as follows:

to differentiate a function of y with respect to x, we differentiate with respect to y and then multiply by $\frac{dy}{dx}$.

Key Point

$$\frac{d}{dx}(f(y)) = \frac{d}{dy}(f(y)) \times \frac{dy}{dx}$$

$$\frac{df}{dx} = \frac{df}{dy} \cdot \frac{dy}{dx}$$

$$\frac{df}{dx} = \frac{df}{dy}\, y'$$

Oct 28-7:42 AM

Implicit Differentiation Process

Implicit Differentiation Process

1. Differentiate both sides of the equation with respect to x.
2. Collect the terms with y' on one side of the equation.
3. Factor out y'.
4. Solve for y'.

Oct 28-7:42 AM

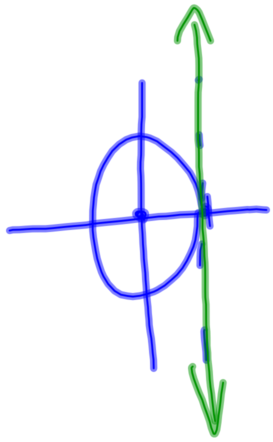
Implicit Differentiation

Find y'. $1 = \frac{x^2}{4} + \frac{y^2}{16}$

$0 = \frac{2x}{4} + \frac{2y\,y'}{16}$

$\frac{8}{y} \cdot -\frac{1}{2}x = \frac{2y}{16}\,y'$

$-4x/y = y'$



Oct 6-9:10 PM

Implicit Differentiation

Find $\frac{dy}{dx}$. $5x^3 - \frac{1}{4} + xy^2 = 7y$

$15x^2 - 0 + x2yy' + (1)y^2 = 7y'$

$15x^2 + y^2 = 7y' - 2xyy'$

$\frac{15x^2 + y^2}{7-2xy} = \frac{y'(7-2xy)}{7-2xy}$

Oct 6-9:10 PM

Implicit Differentiation

Find $\frac{dy}{dx}$. $x\sin(y) + y\cos(x) = 0$

$$\left(x\cos(y)y' + \sin(y)\right) + \left(y(-\sin(x)) + y'\cos(x)\right) = 0$$

$$\sin(y) - y\sin(x) = -x\cos(y)y' - y'\cos(x)$$

$$\frac{\sin(y) - y\sin(x)}{-x\cos y - \cos x} = y'$$

Oct 6-9:10 PM

Implicit Differentiation

Find y' at ($\frac{1}{4}$, 2). $16x^2 + xy^2 = 2$

$$32x + x2yy' + y^2 = 0$$

$$\frac{32x + y^2}{-2xy} = y'$$

$$\frac{32(\frac{1}{4}) + 2^2}{-2(\frac{1}{4})2} = y'$$

$$32(\tfrac{1}{4}) + \tfrac{1}{4} \cdot 2 \cdot 2 y' + 4 = 0$$

$$y' + 8 + 4 = 0$$

$$y' = -12$$

Oct 6-9:10 PM

Implicit Differentiation

Find the equation of the tangent line at (1, 2).

$$\frac{xy}{4} + \frac{1}{y} = x \quad \left(\frac{1}{y} \rightarrow y^{-1}\right)$$

$$\frac{1}{4}(xy' + y) + (-1)y^{-2}y' = 1$$

$$xy' + y - 4y^{-2}y' = 4 - y$$

$$\frac{y'(x - 4y^{-2})}{x - 4y^{-2}} = \frac{4-y}{x-4y^{-2}}$$

$$@(1,2) \quad y' = \frac{4-2}{1-4(2)^{-2}} = \frac{2}{0} \rightarrow \text{vertical tangent}$$

$x = 1$

Oct 6-9:10 PM

Circles

There is a circle centered at (4, 0) with a radius of 13. What is the slope of the tangent line(s) at x = -1?

$$(x-4)^2 + y^2 = 13^2 \rightarrow 25 + y^2 = 169$$

$$y^2 = 144$$

$$y = \pm 12$$

$$2(x-4) + 2yy' = 0$$

$$\frac{2(x-4)}{-2y} = \frac{-2yy'}{-2y}$$

$$\frac{4-x}{y} = y'$$

$(-1, 12)$: $y' = \frac{5}{12}$

$(-1, -12)$: $y' = \frac{5}{-12}$

Oct 6-9:10 PM

Implicit Differentiation

Where is the slope of the curve defined?

$$\frac{xy}{4} + y^2 = x$$

$$4\left(\frac{xy' + y}{4} + 2yy' = 1\right)$$

$$xy' + y + 8yy' = 4 - y$$

$$\frac{y'(x+8y)}{x+8y} = \frac{4-y}{x+8y}$$

$$y' = \frac{4-y}{x+8y} \rightarrow x+8y \neq 0$$

$$8y \neq -x$$

$$y \neq \frac{-x}{8}$$

Oct 6-9:10 PM

Second Derivatives

Find y' and y''.

$$x^4 - 3y^2 + \sin(y) = 3$$

$$4x^3 - 6yy' + \cos(y)y' = 0$$

$$y'(\cos y - 6y) = -4x^3$$

$$y' = \frac{-4x^3}{\cos y - 6y}$$

$$y'' = \frac{(\cos y - 6y)(-12x^2) + 4x^3(-\sin(y)y' - 6y')}{(\cos y - 6y)^2}$$

$$y'' = \frac{(\cos y - 6y)(-12x^2) + 4x^3\left(-\sin y\left(\frac{-4x^3}{\cos y - 6y}\right) - 6\left(\frac{-4x^3}{\cos y - 6y}\right)\right)}{(\cos y - 6y)^2}$$

Oct 6-9:10 PM

Rational Powers

RULE 9 Power Rule for Rational Powers of *x*

If n is any rational number, then

$$\frac{d}{dx}x^n = nx^{n-1}.$$

If $n < 1$, then the derivative does not exist at $x = 0$.

Oct 6-9:10 PM

Proof of Rational Powers Rule

Find y'. $y = x^{p/q}$; with p and q integers, q>0

$(y)^q = (x^{p/q})^q$

$y^q = x^p$

$\frac{qy^{q-1}y'}{qy^{q-1}} = \frac{px^{p-1}}{qy^{q-1}} \Rightarrow y' = \frac{p}{q} \cdot \frac{x^{p-1}}{y^{q-1}}$

$y' = \frac{p}{q} \cdot \frac{x^{p-1}}{(x^{p/q})^{q-1}}$

$y' = \frac{p}{q} \frac{x^{p-1}}{x^{p-p/q}}$

$= \frac{p}{q} x^{(p-1)-(p-\frac{p}{q})}$

$y' = \frac{p}{q} x^{\frac{p}{q}-1}$

Oct 28-8:48 AM

Chain Rule Extension

If n is a rational number and u is a differentiable function of x, then u^n is a differentiable function of x and

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx},$$

provided that $u \neq 0$ if $n < 1$.

Oct 28-8:55 AM

Chain Rule Review

Find y'. $y = (7x - 3)^5$

$y' = 5(7x-3)^4(7)$

Oct 6-9:10 PM

Chain Rule Review

Find y'. $y = [\sec(x) + x^{-3}]^{1.5}$

$1.5[\sec x + x^{-3}]^{0.5}(\sec x \tan x - 3x^{-4})$

Oct 6-9:10 PM

Homework

Page 167 - 169

1 - 43 odd, 59 - 64 all

May 13-10:02 PM