Math woooo!

AP Calculus

Chapter 4

Section 4-3

May 13-10:02 PM

Inverse Functions

$(f(x))^{-1}$

Find $f^{-1}(x)$. $f(x) = 27x^3 - 1$

$y = 27x^3 - 1$

$x = 27y^3 - 1$

$x + 1 = 27y^3$

$f^{-1}(x) = \sqrt[3]{\frac{x+1}{27}} = y$

$f^{-1}(x) = \frac{1}{3}(x+1)^{1/3}$

Oct 6-9:10 PM

Inverse Functions

$f(x) = 27x^3 - 1$ $h(x) = \frac{1}{3}(x+1)^{1/3}$

Find $f(\frac{1}{3})$. $(\frac{1}{3}, 0)$

$27(\frac{1}{27}) - 1$

0

Find $h(0)$. $(0, \frac{1}{3})$

$\frac{1}{3}(0+1)^{1/3}$

$\frac{1}{3}$

Find $f(-1)$.

-28 $(-1, -28)$

Find $h(-28)$.

-1 $(-28, -1)$

Oct 6-9:10 PM

Inverse Functions

$f(x) = 27x^3 - 1$ $h(x) = \frac{1}{3}(x+1)^{1/3}$

$f'(x) = 81x^2$ $(\frac{1}{3}, 0)$

$h'(x) = \frac{1}{9}(x+1)^{-2/3}(1)$ $(0, 1/3)$

Find $f'(\frac{1}{3})$.

9 $(\frac{1}{3}, 9)$

Find $h'(0)$.

$\frac{1}{9}$ $(0, \frac{1}{9})$

Find $f'(-1)$.

81

Find $h'(-28)$.

$\frac{1}{81}$

Oct 6-9:10 PM

Inverse Functions

Find $f^{-1}(x)$. $f(x) = x^3 + 1$

$y = x^3 + 1$

$x = y^3 + 1$

$f^{-1}(x) = (x-1)^{1/3}$

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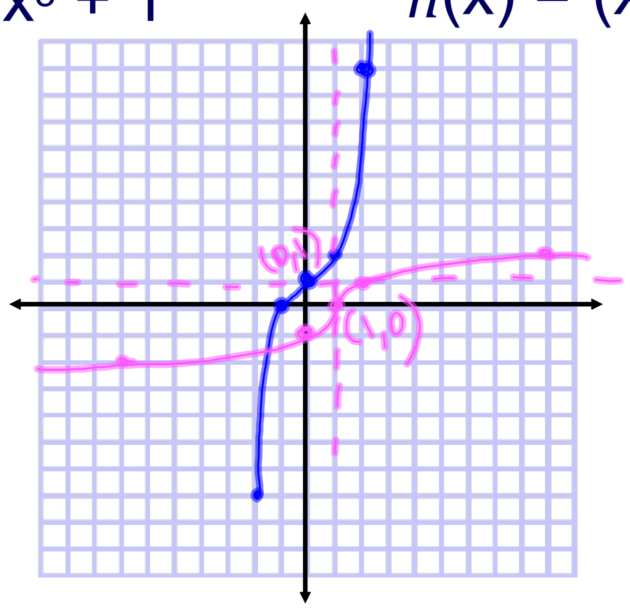
Graphing Inverse Functions

$f(x) = x^3 + 1$ $h(x) = (x - 1)^{1/3}$



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Inverse Function Tables

$f(x) = x^3 + 1$ $h(x) = (x - 1)^{1/3}$

$f'(x) = 3x^2$ $h'(x) = \frac{1}{3}(x-1)^{-2/3}$

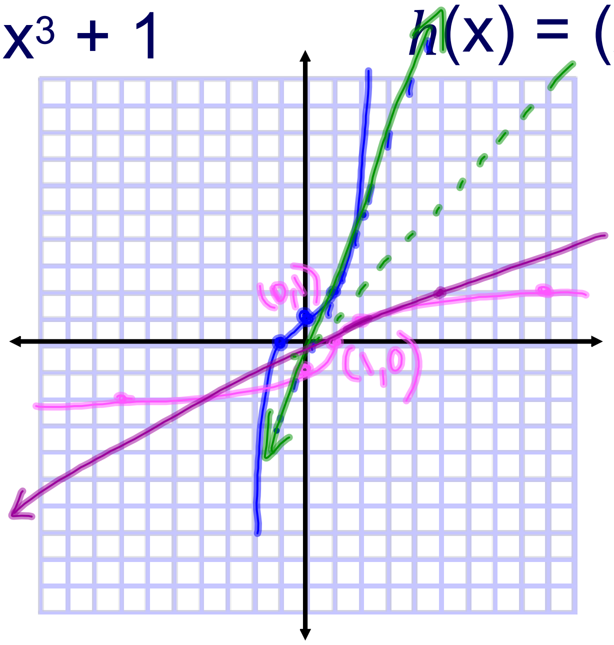
(0,1)
(1,0)

x =	$f(x)$	$h(x)$	$f'(x)$	$h'(x)$
0	1	-1	0	1/3
1	2	0	3	undefined → $\frac{9}{0}$
2	9	1	12	1/3

Oct 6-9:10 PM

Graphing Tangents of Inverse Functions

$f(x) = x^3 + 1$ $h(x) = (x - 1)^{1/3}$



Oct 6-9:10 PM

Let $f(x) = x^5 + 2x - 1$. Since the point $(1, 2)$ is on the graph of f, it follows that the point $(2, 1)$ is on the graph of f^{-1}. Can you find

$$\boxed{\frac{df^{-1}}{dx}(2),}$$

the value of df^{-1}/dx at 2, without knowing a formula for f^{-1}?

1. Graph $f(x) = x^5 + 2x - 1$. A function must be one-to-one to have an inverse function. Is this function one-to-one? $5x^4+2$ ✓
2. Find $f'(x)$. How could this derivative help you to conclude that f has an inverse?

$$f'(1) = \frac{1}{(f^{-1})'(2)}$$

$$f'(1) = 7$$

$$(f^{-1})'(2) = \frac{1}{7}$$

Oct 31-9:08 AM

Inverse Trigonometric Functions

$f(x) = \sin(x)$ $y = \sin x \rightarrow x = \sin^{-1}(y)$

$g(x) = \cot(x)$ $y = \cot x \rightarrow x = \cot^{-1}(y)$

Oct 6-9:10 PM

Inverse Trigonometric Functions

arcsin(x) = $\sin^{-1}(x)$ $\neq \sin(x)^{-1} = \csc(x)$

arccos(x) = $\cos^{-1}(x)$

arctan(x) = $\tan^{-1}(x)$

arcsec(x) = $\sec^{-1}(x)$

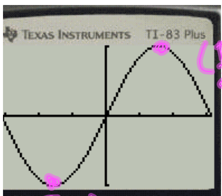
arccsc(x) = $\csc^{-1}(x)$

arccot(x) = $\cot^{-1}(x)$

Oct 6-9:10 PM

Graphing Inverses

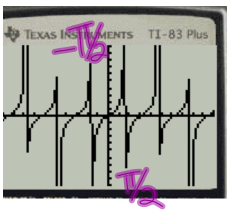
y = sin(x)



$(\frac{\pi}{2}, 1)$

$(-\frac{\pi}{2}, -1)$

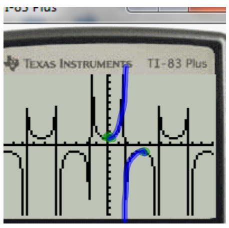
y = tan(x)



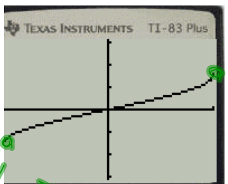
$-\pi/2$

$\pi/2$

y = sec(x)



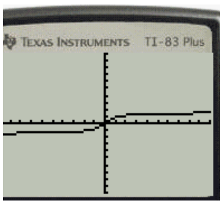
y = $\sin^{-1}(x)$



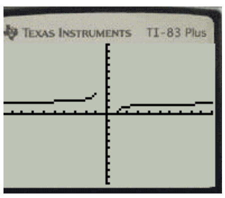
$(1, \frac{\pi}{2})$

$(-1, -\frac{\pi}{2})$

y = $\tan^{-1}(x)$



y = $\sec^{-1}(x)$



Dom:
$[-1, 1]$

Dom:
$(-\infty, \infty)$
Range:
$(-\pi/2, \pi/2)$

Dom:
$(-\infty, -1] \cup [1, \infty)$
Range:
$[0, \pi]$

Oct 6-9:10 PM

Inverse Trigonometric Functions

Find y'. $y = \arcsin(x)$ $y = \sin^{-1}(x)$ Domain: [-1, 1]

$\sin y = x$

$\cos y \cdot y' = 1$

$y' = \frac{1}{\cos y}$

$y' = \frac{1}{\sqrt{1-x^2}}$

$\sin^2 y + \cos^2 y = 1$

$\cos y = \sqrt{1-(\sin(y))^2}$

$\cos y = \sqrt{1-x^2}$

Oct 6-9:10 PM

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

Oct 31-9:11 AM

Inverse Trigonometric Functions

Find y'. $y = \tan^{-1}(x)$ Domain: All Real

$x = \tan y$

$1 = \sec^2 y \cdot y'$

$y' = \frac{1}{\sec^2 y}$

$\sec^2 y = \tan^2 y + 1$

$y' = \frac{1}{(\tan y)^2 + 1}$

$y' = \frac{1}{x^2+1}$

Oct 6-9:10 PM

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$

Oct 31-9:11 AM

Inverse Trigonometric Functions

Find y'. $y = \sec^{-1}(x)$ Domain: $|x| \geq 1$

$x = \sec y$

$1 = \sec y \tan y \; y'$

$y' = \dfrac{1}{\sec y \tan y}$

$y' = \dfrac{1}{x \cdot \pm\sqrt{x^2-1}}$

$\sec^2 y = 1 + \tan^2 y$

$\tan y = \pm\sqrt{(\sec y)^2 - 1}$

$\tan y = \pm\sqrt{x^2-1}$

Oct 6-9:10 PM

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1} x = \begin{cases} +\dfrac{1}{x\sqrt{x^2-1}} & \text{if } x > 1 \\ -\dfrac{1}{x\sqrt{x^2-1}} & \text{if } x < -1 \end{cases}$$

$\rightarrow \dfrac{1}{|x|\sqrt{x^2-1}}$

Oct 31-9:11 AM

Inverse Trigonometric Identities

$$\cos^{-1} x = \pi/2 - \sin^{-1} x$$
$$\cot^{-1} x = \pi/2 - \tan^{-1} x$$
$$\csc^{-1} x = \pi/2 - \sec^{-1} x$$

Oct 31-9:18 AM

Calculator Conversion Identities

$$\sec^{-1} x = \cos^{-1}(1/x)$$

Why different? $\longrightarrow$ $\cot^{-1} x = \pi/2 - \tan^{-1} x$

$$\csc^{-1} x = \sin^{-1}(1/x)$$

Nov 7-8:41 AM

Inverse Trigonometric Functions

$y = \sin^{-1}(x)$

Find y'(x) and y''(x) and simplify.

$$y' = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2}$$

$$y'' = -\cancel{\tfrac{1}{2}}(1-x^2)^{-3/2} \cdot (-\cancel{2}x)$$

$$y'' = x(1-x^2)^{-3/2}$$

If $y''' = x\left(-\tfrac{3}{2}(1-x^2)^{-5/2}(-2x)\right) + 1 \cdot (1-x^2)^{-3/2}$

Oct 6-9:10 PM

Inverse Trigonometric Functions

$y = \arctan(x^2 - 3)$

Find y'(x) and y''(x) and simplify.

$$y' = \frac{1}{1+(x^2-3)^2} \cdot 2x = \frac{2x}{1+(x^2-3)^2}$$

$$y'' = \frac{(1+(x^2-3)^2)(2) - 2x(2(x^2-3)\cdot 2x)}{(1+(x^2-3)^2)^2}$$

Oct 6-9:10 PM

Inverse Trigonometric Functions

$y = \cot^{-1}(x)$

Find the equations for the lines tangent to the function at x=0 and x=1.

$y = \frac{\pi}{2} - \tan^{-1}(x)$

$y' = -\frac{1}{1+x^2}$

$y'(0) = -1$

$\cot^{-1}(0) = \frac{\pi}{2}$

$y - \frac{\pi}{2} = -1(x-0)$

$y'(1) = -\frac{1}{2}$

$\cot^{-1}(1) = \frac{\pi}{4}$

$y - \frac{\pi}{4} = -\frac{1}{2}(x-1)$

Oct 6-9:10 PM

Inverse Trigonometric Functions

$y = \text{arcsec}(x) = \sec^{-1}(x)$

Find the equations for the lines tangent to the function at x=2 and x=-5. (Round to the thousandths if necessary)

$y' = \frac{1}{|x|\sqrt{x^2-1}}$

$y'(2) = \frac{1}{2\sqrt{4-1}} = \frac{1}{2\sqrt{3}}$

$\sec^{-1}(2) = \frac{\pi}{3}$

$y - \pi/3 = \frac{1}{2\sqrt{3}}(x-2)$

$y'(-5) = \frac{1}{5\sqrt{24}}$

$\sec^{-1}(-5) = \cos^{-1}(-1/5) \approx 1.772$

$y - 1.772 = \frac{1}{5\sqrt{24}}(x+5)$

Oct 6-9:10 PM

Homework

Page 175

1 - 13 odd, 27 - 29 all, 31 - 40 all, 48

May 13-10:02 PM