AP Calculus

Chapter 4

Section 4-4

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Review: Rules of Exponentials

Solve for y.

$x = e^y$

$\ln x = \ln e^y$

$\ln x = y$

$\ln = \log_e$

$\log_e e = 1$ ∴

$x = 7 + k^{y+1}$

$x - 7 = k^{y+1}$

$\log_k (x-7) - 1 = y$

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Review: Rules of Logarithms

Solve for y.

$x = \ln y$ $\quad x = \ln b^y$

$x = \log_4 (y^2 - 1)$

$e^x = y$ $\quad e^x = b^y$

$4^x = y^2 - 1$

$\pm\sqrt{4^x + 1} = y$

$\log_b e^x = y$

$\frac{\ln e^x}{\ln b} = \frac{x}{\ln b} = y$

or $x = y \ln b$

$\frac{x}{\ln b} = y$

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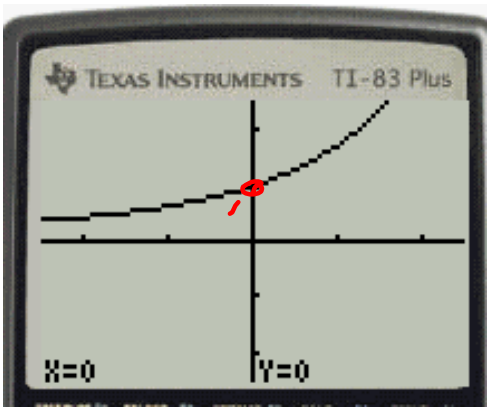
Exponential Functions

Find $\lim_{x \to 0} \frac{e^x - 1}{x}$ graphically.



$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

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Exponential Functions Derivative

Calculate $f'(x)$. $f(x) = e^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$$

$$\lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} = e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$$

$$\frac{d}{dx}(e^x) = f'(x) = e^x$$

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Exponential Functions Derivative

Calculate y'. $y = e^{u(x)}$

- $y' = e^{u(x)} \cdot u'(x)$

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Exponential Functions Derivative

Calculate y′. $y = 2.5e^{(x^2 + 6x)}$

$y' = 2.5e^{(x^2+6x)} \cdot (2x+6)$

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Exponential Functions Derivative

Calculate y′. $y = xe^{\cos(x)}$

$y' = x \cdot e^{\cos x} \cdot (-\sin x) + e^{\cos x} \cdot 1$

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Exponential Functions Derivative

Calculate y′. $y = a^x$

$a^x = y = e^{x \ln a}$

$\ln y = \ln a^x$

$\log_e y = \ln y = x \ln a$

$y = e^{x \cdot \ln a}$

$y' = e^{x \ln a} \cdot \ln a$

$y' = a^x \cdot \ln a$

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Exponential Functions Derivative

Calculate y′. $y = 7^x$

$y' = 7^x \cdot \ln 7$

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Exponential Functions Derivative

Calculate y′. $y = x^2 + 5^x$

$y' = 2x + 5^x \ln 5$

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Exponential Functions Derivative

Calculate y′. $y = b^{p(x)}$

$y' = b^{p(x)} \cdot \ln b \cdot p'(x)$

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Exponential Functions Derivative

Calculate y′. $y = 21^{7x+\sqrt{x}}$

$y' = 21^{7x+\sqrt{x}} \cdot \ln 21 \cdot \left(7 + \frac{1}{2}x^{-1/2}\right)$

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Exponential Functions Derivative

Calculate y′. $y = \ln x$

$e^y = x$

$e^y \cdot y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{x} = x^{-1}$

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Exponential Functions Derivative

Calculate y′. $y = \ln(x^4) = 4 \cdot \ln x$

$y' = \frac{1}{x^4} \cdot 4x^3 = \frac{4}{x}$

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Exponential Functions Derivative

Calculate y′. $y = \ln(u(x))$

$y' = \frac{1}{u(x)} \cdot u'(x)$

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Exponential Functions Derivative

Calculate $y'\left(\frac{\pi}{2}\right)$. $y = \ln x + \ln(\sin x)$

$$y' = \frac{1}{x} + \frac{\cos x}{\sin x}$$

$$y' = \frac{1}{x} + \cot x$$

$$y'\left(\frac{\pi}{2}\right) = \frac{2}{\pi} + 0 = \frac{2}{\pi}$$

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Exponential Functions Derivative

Calculate y'.

**Use change of base

$y = \log_a x$

$$y = \frac{\ln x}{\ln a} = \frac{1}{\ln a} \cdot \ln x$$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}$$

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Exponential Functions Derivative

Calculate y′. $y = \log_6 (x^2 - 15x + 7)$

$$y' = \frac{1}{(x^2+15x+7)\ln 6} \cdot (2x-15)$$

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All Real Power Rule

$$y = x^{\pi}$$

Show that $y' = \pi x^{\pi-1}$.

$$\ln y = \ln x^{\pi}$$

$$\ln y = \pi \ln x$$

$$\frac{1}{y} \cdot y' = \frac{\pi}{x}$$

$$y' = \frac{\pi \cdot y}{x} = \pi \frac{x^{\pi}}{x^1} = \pi x^{\pi-1}$$

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RULE 10 Power Rule for Arbitrary Real Powers

If u is a positive differentiable function of x and n is any real number, then u^n is a differentiable function of x, and

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}.$$

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$$\frac{d}{dx}e^u = e^u\frac{du}{dx}.$$

For $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}.$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}.$$

For $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}.$$

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Homework

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