1)

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

Write an equation for the line tangent to the curve at the point $(-2, 1)$.

$$2x + 2 + 4y^3 y' + 4y' = 0$$
$$2(-2) + 2 + 4(1)^3 y' + 4y' = 0$$
$$-4 + 2 + 4y' + 4y' = 0$$
$$8y' = 2$$
$$y' = 1/4$$

$$y - 1 = 1/4\,(x+2)$$

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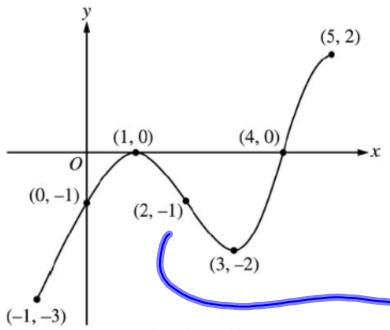

Graph of f'

$\rightarrow f'(2) = -1$

The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

Let g be the function defined by $g(x) = x f(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

$$g'(x) = x f'(x) + f(x)$$
$$g'(2) = 2f'(2) + f(2)$$
$$= 2(-1) + 6$$
$$= 4$$

$$g(2) = 2 \cdot 6 = 12$$

$$y - 12 = 4(x-2)$$

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4)

$\lim_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

$$\lim_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{h}\right) = \lim_{h\to 0} \frac{\ln(2+h)-\ln 2}{h} \Rightarrow \frac{d}{dx}(\ln x) @ x=2$$

$$\hookrightarrow \frac{1}{x} @ x=2$$

$$1/2$$

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5)

$\lim_{x\to 0}(x\csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

$$\lim_{x\to 0} x\csc x = \lim_{x\to 0} \frac{x}{\sin x}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

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6)

If $xy^2 + 2xy = 8$, then, at the point $(1, 2)$, y' is ...

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

$$x \cdot 2yy' + y^2 + 2xy' + 2y = 0$$

plug in (1,2)

$$4y' + 4 + 2y' + 4 = 0$$

$$6y' + 8 = 0$$

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7)

If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

$$2x + 2yy' = 0$$

$$y' = \frac{-x}{y} \rightarrow y'' = \frac{y(-1) - (-x)(y')}{y^2}$$

$$= \frac{-4}{3} \qquad y'' = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$\frac{-3 + 4\left(-\frac{4}{3}\right)}{9}$$

$$\frac{-\frac{9}{3} - \frac{16}{3}}{9} = \frac{-25}{27}$$

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8)

If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

$$\sec^2(xy)(xy' + y) = 1$$

$$\frac{1}{\cos^2(xy)}(xy' + y) = 1$$

$$xy' + y = \cos^2(xy)$$
$$-y \qquad\qquad -y$$

$$\frac{xy'}{x} = \frac{\cos^2(xy) - y}{x}$$

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9) $f(v) = v^2 + 2v$

If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$ $\rightarrow f'(\ln x) \cdot \frac{1}{x}$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

$$(\ln x)^2 + 2\ln x$$

$$2\ln x \cdot \frac{1}{x} + \frac{2}{x}$$

$$f'(v) = 2v + 2$$

$$[2(\ln x) + 2]\frac{1}{x}$$

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10)

$u = 2/x = 2x^{-1}$

If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

$e^{2/x} \cdot -2x^{-2}$

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11)

If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

$-\sin(3x) \cdot 3$

$-\sin\left(\frac{\pi}{3}\right) \cdot 3 = \frac{-3\sqrt{3}}{2}$

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12)

If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

$(x-1)\,3\cancel{(x^2+2)^2}(2x) + (x^2+2)^3(1)$

$$(x^2+2)^2\left(\begin{array}{c} \overset{6x}{3(x-1)(2x)} + (x^2+2) \\ 6x^2-6x+x^2+2 \\ 7x^2-6x+2 \end{array}\right)$$

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13)

$f(1)=2 \quad (1,2)$

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

$g'(2) = \frac{1}{4}$

$f'(1) = 3x^2 + 1 = 4$

1

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14)

If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

$$\frac{1}{x+4+e^{-3x}}\left(1+0-3e^{-3x}\right)$$

$$\frac{1}{5}(-2)$$

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15)

If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

$$2(x^3+1)(3x^2)$$

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16)

If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$ $\cos(e^{-x}) \cdot e^{-x} \cdot -1$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

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17)

If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

$$f(x) = e^{3\ln x^2} = e^{\ln x^6} = x^6$$

$$f'(x) = 6x^5$$

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18)

If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$$\frac{2}{3}(x^2-2x-1)^{-1/3} \cdot \cancel{2}(x-2)$$

$$\left(-2/3\right)\ (-2) = \frac{4}{3}$$

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19)

If $f(x) = \frac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\frac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\frac{e^{2x}(2x+1)}{x^2}$

(E) $\frac{e^{2x}(2x-1)}{2x^2}$

$$\frac{2xe^{2x}\cdot 2 - e^{2x}\cdot 2}{(2x)^2}$$

$$\frac{\cancel{2}e^{2x}(2x-1)}{2\cancel{4}x^2}$$

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20)

$\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

$(\cos(x^3))^2$

$2\cos(x^3) \cdot (-\sin(x^3))\, 3x^2$

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