AP Calculus

Chapter 5

Section 5-1

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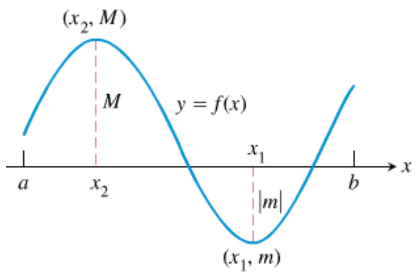
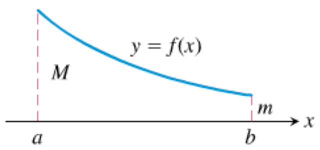
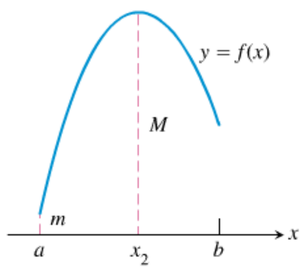
Maximum and Minimum Values



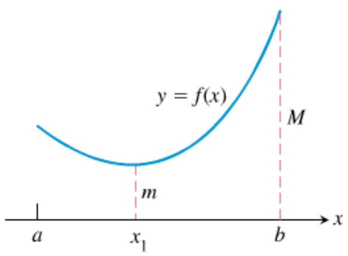
Maximum and minimum at interior points



Maximum and minimum at endpoints



Maximum at interior point, minimum at endpoint



Minimum at interior point, maximum at endpoint

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DEFINITION Absolute Extreme Values

Let f be a function with domain D. Then $f(c)$ is the

(a) **absolute maximum value** on D if and only if $f(x) \leq f(c)$ for all x in D.
(b) **absolute minimum value** on D if and only if $f(x) \geq f(c)$ for all x in D.

Absolute (or **global**) maximum and minimum values are also called **absolute extrema** (plural of the Latin *extremum*). We often omit the term "absolute" or "global" and just say maximum and minimum.

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Use the parent function $y = x^2$ and algebraic transformations to find any absolute extrema.

$$y = -2(x - 3)^2 + 4$$

Max: 4 @ x=3

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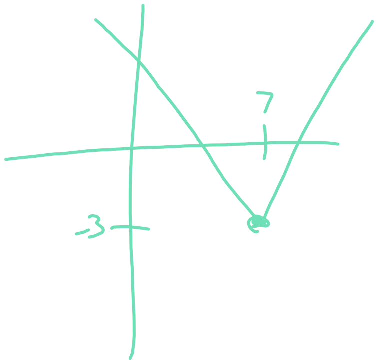
Use the parent function $y = |x|$ and algebraic transformations to find any absolute extrema.

$$y = |x - 7| - 3$$

min: -3 @ $x=7$



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DEFINITION Local Extreme Values

Let c be an interior point of the domain of the function f. Then $f(c)$ is a

(a) **local maximum value** at c if and only if $f(x) \le f(c)$ for all x in some open interval containing c.

(b) **local minimum value** at c if and only if $f(x) \ge f(c)$ for all x in some open interval containing c.

A function f has a local maximum or local minimum *at an endpoint c* if the appropriate inequality holds for all x in some half-open domain interval containing c.

Local extrema are also called **relative extrema.**

An **absolute extremum** is also a local extremum, because being an extreme value overall makes it an extreme value in its immediate neighborhood. Hence, *a list of local extrema will automatically include absolute extrema if there are any.*

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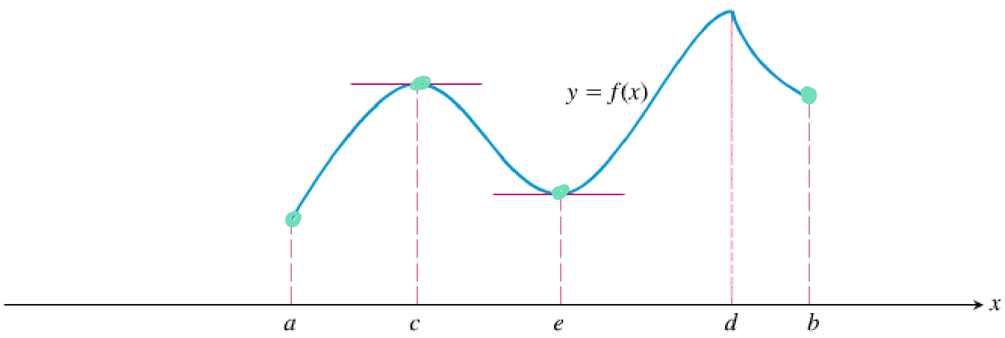

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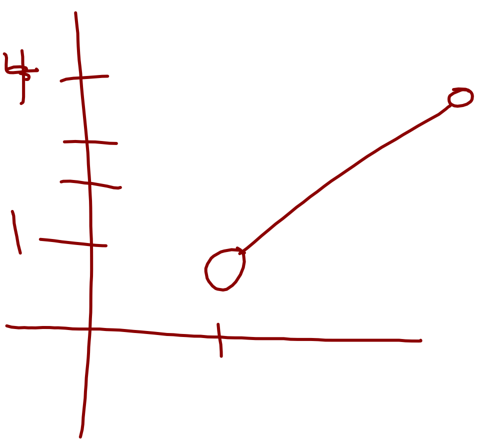
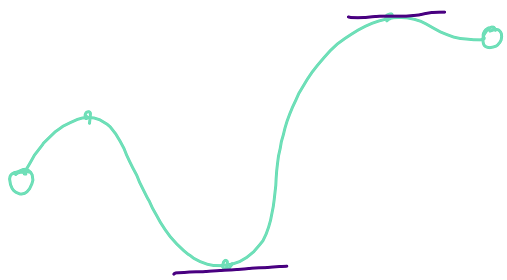
THEOREM 1 The Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval. (Figure 5.3)

Why must the interval be closed for the theorem to hold true?





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DEFINITION Critical Point

A point in the interior of the domain of a function f at which $f' = 0$ or f' does not exist is a **critical point** of f.

Thus, in summary, extreme values occur only at critical points and endpoints.

DEFINITION Stationary Point

A point in the interior of the domain of a function f at which $f' = 0$ is called a ***stationary point*** of f.

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Find all critical values of the function.

$$y = 1.2x^5 - 2x^3$$

$$y' = 6x^4 - 6x^2$$

$$0 = 6x^4 - 6x^2$$

$$0 = 6x^2(x^2 - 1)$$

$$x = \pm 1, 0$$

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Find all critical values of the function.

$$y = x^{2/3}$$

$$y' = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$x = 0$$

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Find all critical values. Identify all critical values that are stationary points.

$$y = \sqrt{x^3 - 3x}$$

$$y' = \frac{1}{2}(x^3 - 3x)^{-1/2}(3x^2 - 3)$$

$$0 = \frac{3x^2 - 3}{2\sqrt{x^3 - 3x}}$$

$3x^2 - 3 = 0$

$3x^2 = 3$

$x = \pm 1$

Stationary

$2\sqrt{x^3 - 3x} = 0$

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0 \quad x = \pm\sqrt{3}$

Not stationary

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THEOREM 2 Local Extreme Values

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0.$$

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Find all relative and absolute extrema of the function

$$y = 2x^3 - x^4 + 2x^2$$

$$y' = 6x^2 - 4x^3 + 4x$$

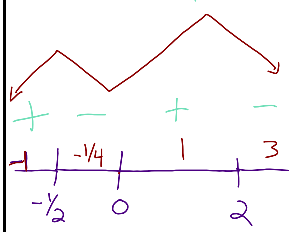
$$0 = 6x^2 - 4x^3 + 4x$$

$$0 = -2x(2x^2 - 3x - 2)$$

$x = 0$ $\quad$ $2x^2 - 3x - 2 = 0$

$$(2x+1)(x-2) = 0$$

$$x = -\tfrac{1}{2},\ 2$$



$$y' = \overline{-2x}(\overline{2x+1})(\overline{x-2})$$

$y'(-1) = + \cdot - \cdot -$

$y'(-1/4) = +(+)(-)$

$y'(1) = -(+)(-)$

$y'(3) = -(+)(+)$

max @ $-\tfrac{1}{2}, 2$

min @ 0

$y(-\tfrac{1}{2}) = 3/16$ rel max

$y(2) = 8$ abs max

$y(0) = 0$ rel min

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Find all relative and absolute extrema of the function

$$y = x^4 - \frac{8}{3}x^3 - 8x^2 + 32x + 12$$

$4x^3 - 8x^2 - 16x + 32 = 0$

$4(x^3 - 2x^2 - 4x + 8)$

$4(x^2(x-2) - 4(x-2))$

$4(x^2-4)(x-2)$

$y' = 4(x+2)(x-2)^2\cancel{(x-2)} = 0$

$x = 2, -2$

$-$ $+$ $+$

-3 0 3

-2 2

$f(-2) = (-2)^4 - \frac{8}{3}(-2)^3 - 8(-2)^2 + 32(-2) + 12$

abs min

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Find all relative and absolute extrema of the function

$$y = \frac{25}{6}x^{1.2} - 10x + 46$$

$6/5$

$y' = \frac{25}{6} \cdot \frac{6}{5} x^{1/5} - 10 = 0$

$5x^{1/5} = 10$

$x^{1/5} = 2$

$x = 2^5 = 32$

$y' = 5x^{1/5} - 10$

$-$ $+$

1^5 2^5 3^5

abs min @ $x = 32$

$f(32) = \frac{25}{6}(32)^{1.2} - 10(32) + 46$

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Find all relative and absolute extrema of the function

$$y = (x^2 - 9)^{1/5}$$

$$y' = \frac{1}{5}(x^2-9)^{-4/5}(2x)$$

$$\frac{2x}{5(x^2-9)^{4/5}} = 0$$

$2x=0$

$x=0$

$5(x^2-9)^{4/5}=0$

$x^2-9=0$

$x=\pm 3$

$-\quad -\quad +\quad +$

$-4\quad -1\quad 1\quad 4$

$-3\quad 0\quad 3$

@ $x=0$ abs min

$f(0)=\sqrt[5]{-9}$

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To the nearest thousandth, find all relative and absolute extrema of the function

$$y = \frac{x-1}{6^x}$$

$$y' = \frac{6^x(1)-(x-1)6^x \cdot \ln 6}{(6^x)^2} = \frac{\cancel{6^x}(1-(x-1)\ln 6)}{\cancel{6^x} \cdot 6^x}$$

$$y' = \frac{1+\ln 6 - x\ln 6}{6^x} = 0$$

$1+\ln 6 - x\ln 6 = 0$

$\frac{1+\ln 6}{\ln 6} = x\cancel{\ln 6}$

$\frac{1}{\ln 6}+1=x$

$+\quad -$

$1\quad 10$

$\frac{1}{\ln 6}+1$

abs max @ $x \approx 1.558$

$f(x) \approx .034$

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Homework

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1 - 39 odd, 43 - 53 all

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