AP Calculus

Chapter 5

Section 5-2

May 13-10:02 PM

You have just arrived home after a long road trip, a fifty-five-mile drive that took you forty minutes, and your mom is ticked off. She screams, "I know you were speeding. You must have been driving over eighty miles-per-hour." Is she correct?

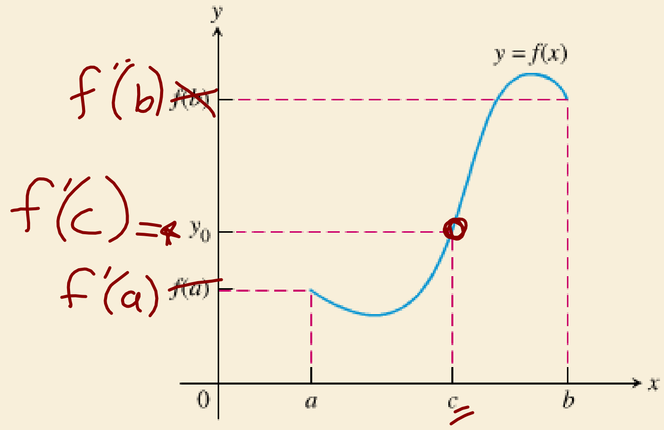
$$\frac{55}{2/3}\ \frac{\text{miles}}{\text{hour}} = \frac{165}{2} = 82.5 \text{ mph}$$

Yes

Dec 11-1:04 PM

THEOREM 8 The Intermediate Value Theorem for Continuous Functions

A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



Sep 17-8:14 AM

THEOREM 3 Mean Value Theorem for Derivatives

cont.

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

diff

slope of secant

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Why do the conditions matter?

- continuous on [a,b]
- differentiable on (a,b)

Dec 11-12:50 PM

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of its interior (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

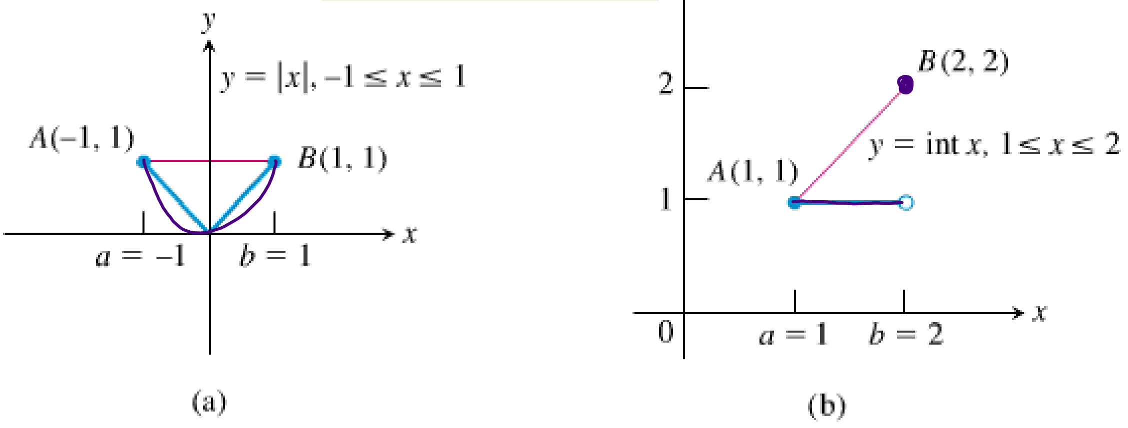


Figure 5.11 No tangent parallel to chord AB.

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Describe how the following is consistent with the MVT. Then use the MVT for the function $f(x) = 2x - 9x^{-1}$ to find the value, c, in the interval (1, 3) so that $\boxed{f'(c) = 5}$. $= 2 + 9x^{-2}$

(a, b)

$\frac{3}{9} = \frac{9x^{-2}}{9}$

$\sqrt{\frac{1}{3}} = \sqrt{\frac{1}{x^2}}$

$\sqrt{\frac{1}{3}} = \frac{1}{x}$

$x = \frac{1}{\sqrt{\frac{1}{3}}}$

$$\frac{f(b) - f(a)}{b - a}$$

$$\frac{(2(3) - 9(3)^{-1}) - (2 - 9)}{2}$$

$$\frac{(6 - 3) - (-7)}{2} = \frac{10}{2} = 5$$

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Describe how the following is consistent with the MVT. Then use the MVT for the function $f(x) = 3\sin(x)$ to show that there is some value, c, in the interval $(0, \frac{3}{2}\pi)$ so that $f'(c) = -\frac{2}{\pi}$.

$f'(x) = 3\cos x = -\frac{2}{\pi}$

$\cos x = -\frac{2}{3\pi}$

$x = \cos^{-1}\left(-\frac{2}{3\pi}\right)$

$\frac{f(a)-f(b)}{a-b}$ $\frac{3}{-\frac{3}{2}\pi}$

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Find the average rate of change for the function $f(x) = \sqrt{x}$ over the interval [0, 9]. Then use the MVT to show that there is some value, c, in (0, 9) so that $f'(c)$ is equal to the average ROC.

$\frac{f(9)-f(0)}{9-0} = \frac{3-0}{9-0} = \frac{1}{3}$

$\sqrt{x}$ cont. on $[0,9]$

$\sqrt{x}$ diff on $(0,9)$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = \frac{1}{3}$

$3 = 2\sqrt{x}$

$\frac{3}{2} = \sqrt{x}$

$x = 9/4$

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DEFINITIONS Increasing Function, Decreasing Function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. f **increases** on I if $x_1 < x_2 \quad \Rightarrow \quad f(x_1) < f(x_2)$.
2. f **decreases** on I if $x_1 < x_2 \quad \Rightarrow \quad f(x_1) > f(x_2)$.

COROLLARY 1 Increasing and Decreasing Functions

Let f be continuous on $[a, b]$ and differentiable on (a, b).

1. If $f' > 0$ at each point of (a, b), then f increases on $[a, b]$.
2. If $f' < 0$ at each point of (a, b), then f decreases on $[a, b]$.

Nov 26-11:30 AM

Find x-coordinates of all local extrema and determine the intervals on which the function is increasing.

$$y = \tfrac{1}{6}x^3 + x^2 - 6x$$

$$y' = \tfrac{1}{2}x^2 + 2x - 6$$

$$0 = \tfrac{1}{2}x^2 + 2x - 6$$

$$0 = x^2 + 4x - 12$$

$$0 = (x-2)(x+6)$$

$$x = 2, -6$$

+ − +

-7 -6 0 2 3

$$\boxed{(-\infty, -6] \cup [2, \infty)}$$

Nov 17-9:24 PM

Find x-coordinates of all local extrema and determine the intervals on which the function is increasing.

$$y = (2x - x^2)^{\frac{1}{2}}$$

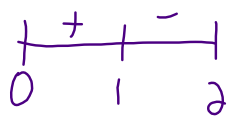
Dom: $[0,2]$

$$y' = \frac{1}{2}(2x-x^2)^{-1/2} \cdot (2-2x)$$

$$0 = \frac{1-x}{\sqrt{2x-x^2}}$$

$1-x=0$
$x=1$

$2x-x^2=0$
$(2-x)x=0$
$x=2, x=0$



$x=1$

$[0,1]$

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Homework

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1 - 8 all, 11, 13, 15 - 27 odd, 39 - 42 all, 52, 53, 54, 56

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