AP Calculus

Chapter 5

Section 5-3

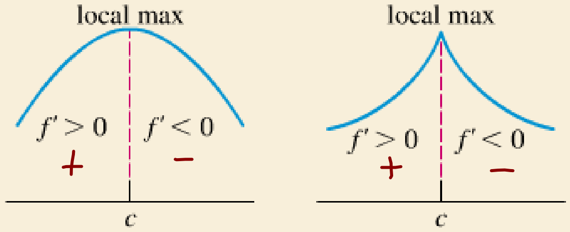
May 13-10:02 PM

THEOREM 4 First Derivative Test for Local Extrema

The following test applies to a continuous function $f(x)$.

At a critical point *c*:

1. If f' changes sign from positive to negative at c ($f' > 0$ for $x < c$ and $f' < 0$ for $x > c$), then f has a local maximum value at c.



(a) $f'(c) = 0$ (b) $f'(c)$ undefined

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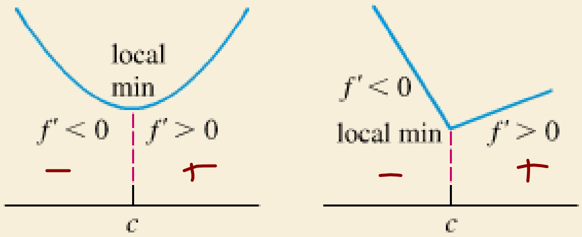
2. If f' changes sign from negative to positive at c ($f' < 0$ for $x < c$ and $f' > 0$ for $x > c$) then f has a local minimum value at c.



(a) $f'(c) = 0$ (b) $f'(c)$ undefined

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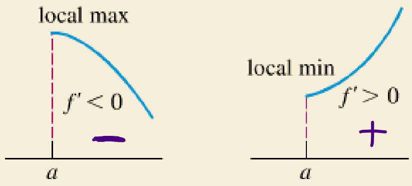
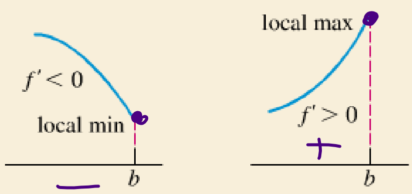
At a left endpoint *a*:

If $f' < 0$ ($f' > 0$) for $x > a$, then f has a local maximum (minimum) value at a.



At a right endpoint *b*:

If $f' < 0$ ($f' > 0$) for $x < b$, then f has a local minimum (maximum) value at b.



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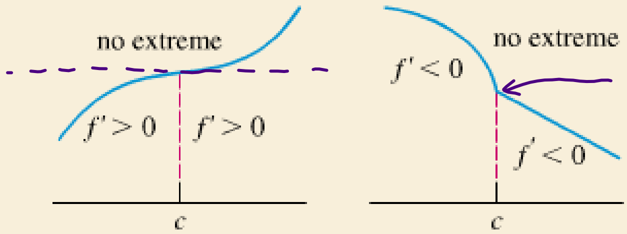
3. If f' does not change sign at c (f' has the same sign on both sides of c), then f has no local extreme value at c.



(a) $f'(c) = 0$ (b) $f'(c)$ undefined

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Determine where all local and absolute extrema occur on the interval

$y = -2x^7 + 7x^4 - 14x + 5$ [-2, 5]

$y' = -14x^6 + 28x^3 - 14$

$y' = -14(x^6 - 2x^3 + 1) \longrightarrow u^2 - 2u + 1$ $x^3 = u$

$y' = -14(x^3 - 1)^2$ $(u-1)^2$

$x = 1$

abs max @ $x = -2$

abs min @ $x = 5$

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Find all local extrema of the function

$$y = x^4(x-3)^{-1}$$

$$y = \frac{x^4}{x-3}$$

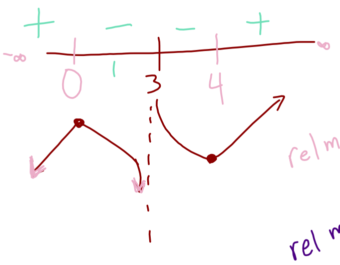
$$y' = -x^4(x-3)^{-2} + (x-3)^{-1}(4x^3)$$

$$y' = \frac{(x-3)(4x^3) - x^4}{(x-3)^2} = \frac{-x^4}{(x-3)^2} + \frac{4x^3}{x-3}\frac{(x-3)}{(x-3)}$$

$$y' = \frac{4x^4 - 12x^3 - x^4}{(x-3)^2}$$

$$y' = \frac{3x^4 - 12x^3}{(x-3)^2} \rightarrow 3x^3(x-4) \quad x = 0, 4$$

$\leftarrow x = 3$ DNE



rel max @ $x=0$

0

rel min @ $x=4$

$4^4 = 256$

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Determine where all local extrema occur on the function

$$y = x^{1.6}(x+15)^{1.4}$$

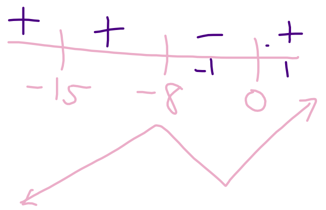
$8/5 \quad 7/5$

$$y' = x^{8/5} \cdot \tfrac{7}{5}(x+15)^{2/5} + (x+15)^{7/5} \cdot \tfrac{8}{5}x^{3/5}$$

$$y' = x^{3/5}(x+15)^{2/5}\left(x \cdot \tfrac{7}{5} + \cancel{(x+15)\tfrac{8}{5}}\right) \quad + \tfrac{8}{5}x + 24$$

$$y' = x^{3/5}(x+15)^{2/5}(3x + 24)$$

$$x = -8, 0, -15$$



rel max @ $x=-8$

rel min @ $x=0$

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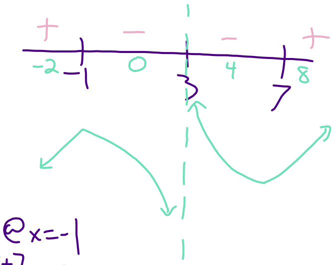
Find any local and absolute extrema of the function

$$y = \frac{x^2 + 7}{x - 3}$$

$$y' = \frac{(x-3)(2x) - (x^2+7)}{(x-3)^2} = \frac{2x^2 - 6x - x^2 - 7}{(x-3)^2}$$

$$y' = \frac{x^2 - 6x - 7}{(x-3)^2} \rightarrow (x-7)(x+1) \rightarrow x = 7, -1 \rightarrow f' = 0$$

$(x-3)^2 \rightarrow x = 3$ DNE



local max @ $x=-1$

$f(-1) = \frac{(-1)^2+7}{-1-3} = -2$

local min @ $x=7$

$f(7) = \frac{7^2+7}{7-3} = \frac{56}{4} = 14$

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DEFINITION Concavity

The graph of a differentiable function $y = f(x)$ is

(a) **concave up** on an open interval I if y' is increasing on I.

(b) **concave down** on an open interval I if y' is decreasing on I.

Concavity Test

The graph of a twice-differentiable function $y = f(x)$ is

(a) concave up on any interval where $y'' > 0$.

(b) concave down on any interval where $y'' < 0$.

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Determine the intervals of concavity for the function

$$y = \frac{1}{3}x^3 + 5x^2 + 25x + 43$$

$y' = x^2 + 10x + 25$

$y'' = 2x + 10$

$0 = 2x + 10$

$x = -5$

$-$ $+$

-6 -4 0

-5

concave down $(-\infty, -5)$

concave up $(-5, \infty)$

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Determine the intervals of concavity for the function

$$y = 4x^5 - 5x^2 - 6x$$

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a) Show that $f(x)$ is continuous
b) Show that $f(x)$ is differentiable
c) Find the location of all extreme values of the function
d) Determine the intervals of concavity for the function

$$f(x) = \begin{cases} 2x^2 - 4x - x + 4, & 1 \le x \le 5 \\ 3x^{4/3} - x^4 - x, & -4 \le x < 1 \end{cases}$$

a) $2(1)^2 - 4(1) - 1 + 4 = 3(1)^{4/3} - 1^4 - 1$

$1 = 1$

b) $4x - 4 - 1 = 4x^{1/3} - 4x^3 - 1$

$4(1) - 4 - 1 = 4(1)^{1/3} - 4(1)^3 - 1$

$-1 = -1$

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a) Show that $f(x)$ is continuous
b) Show that $f(x)$ is differentiable
c) Find the location of all extreme values of the function
d) Determine the intervals of concavity for the function

$$f(x) = \begin{cases} 2x^2 - 4x - x + 4, & 1 \le x \le 5 \\ 3x^{4/3} - x^4 - x, & -4 \le x < 1 \end{cases}$$

$4x - 4 - 1 = 0$

$4x - 5 = 0$

$x = 5/4$

$4x^{1/3} - 4x^3 - 1 = 0$

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a) Show that $f(x)$ is continuous 1
b) Show that $f(x)$ is differentiable 2
c) Find the location of all extreme values of the function 3
d) Determine the intervals of concavity for the function 3

$$f(x) = \begin{cases} 2x^2 - 4x - x + 4, & 1 \le x \le 5 \\ 3x^{4/3} - x^4 - x, & -4 \le x < 1 \end{cases}$$

$y' = 4x - 5$

$y'' = 4$

$4x^{1/3} - 4x^3 - 1$

$\frac{4}{3}x^{2/3} - 12x^2$

− + − +

-4 $-.439$ $.439$ 1 5

Con. up $(-.439, .439) \cup (1, 5)$

Con. down $(-4, -.439) \cup (.439, 1)$

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DEFINITION Point of Inflection

A point where the graph of a function has a $\underbrace{\text{tangent line}}$ and where the concavity changes is a **point of inflection.**

f' exists
or
vertical tangent

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Find the x-coordinate of any points of inflection for the function

$$y = .2x^6 + .5x^5 - x^3 - 5x^2 + .3x - 5.2$$

$y' = 1.2x^5 + 2.5x^4 - 3x^2 - 10x + .3$

$y'' = 6x^4 + 10x^3 - 6x - 10$

$\cancel{2}(3x^4 + 5x^3 - 3x - 5) = 0$

$x^3(3x+5) - 1(3x+5) = 0$

$(x^3 - 1)(3x+5) = 0$

$x = 1$ $x = -5/3$

$+$ $-5/3$ $-$ 1 $+$

POI @ $x = 1, -5/3$

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Find all relative extrema and points of inflection for the function

$$y = 3x^5 - 20x^3$$

$y' = 15x^4 - 60x^2$

$0 = 15x^2(x^2 - 4)$

$15x^2(x+2)(x-2)$

$x = 0, \pm 2$

$+$ -2 $-$ 0 $-$ 2 $+$

rel max @ $x = -2$

$3(-2)^5 - 20(-2)^3$

$y'' = 60x^3 - 120x$

$60x(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

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THEOREM 5 Second Derivative Test for Local Extrema

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

neg

pos

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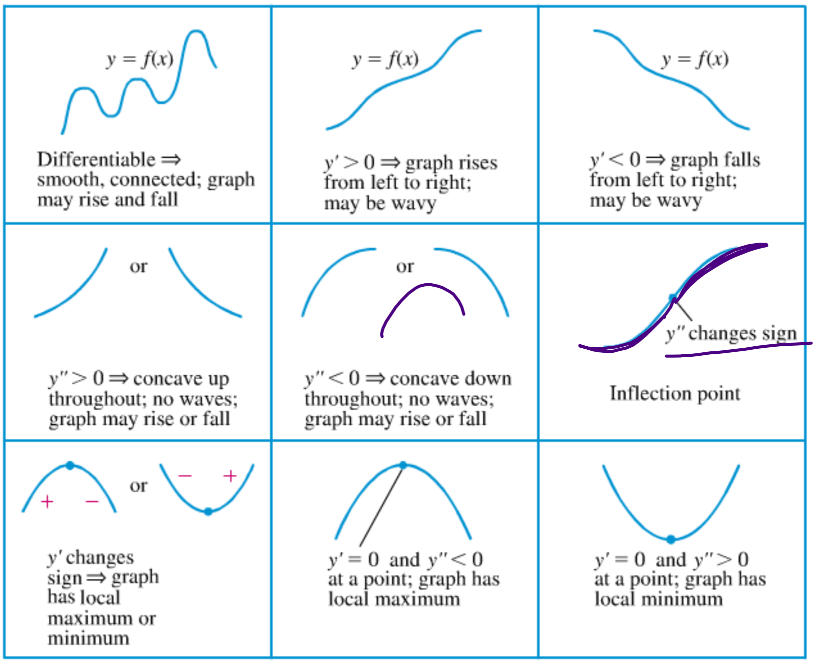

tangent

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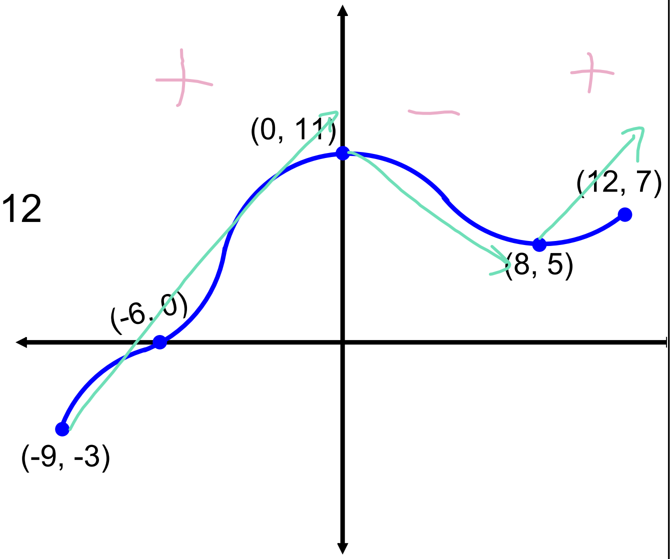
The graph shows $f'(x)$ defined on [-9,12] which has a relative minimum at (8,5) and a relative maximum at (0,11). Find a) where all relative extrema of $f(x)$ occur and b) the intervals of concavity of $f(x)$.



a) relative min at x = -6

relative max at x = -9 and 12

b) concave up on intervals:

(-9,0) and (8,12)

concave down on interval:

(0,8)

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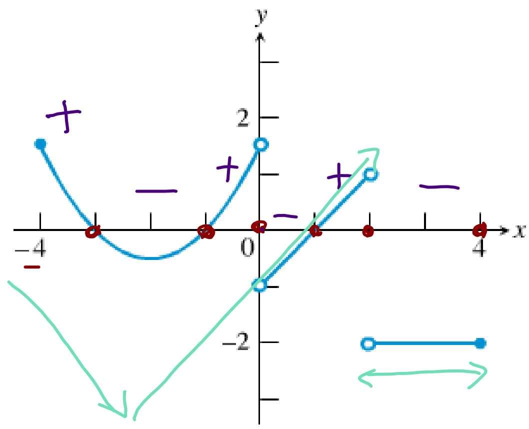
The graph shows $f'(x)$, the derivative of a continuous function, and $f'(x)$ has a relative minimum at x = -2. On [-4,4] find a) where all relative extrema of $f(x)$ occur and b) the intervals of concavity of $f(x)$.



a) rel min @ x = -4, -1, 1, 4

rel max @ x = -3, 0, 2

b) concave up on interval:

(-2,2)

concave down on interval:

(-4,-2)

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Homework

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5 - 14 all, 19 - 25 all, 33 - 43 odd,
51, 52, 53 - 59 odd

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