AP Calculus

Chapter 6

Section 6-2

May 13-10:02 PM

Summation Notation

$$\boxed{\sum_{k=1}^{n} a_k} = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$$

"Σ" means to take the sum of a series of terms

"a" denotes each term in the series

"k" is the number of the term

"k=1" is the starting value for the sum

"n" identifies the last term for the sum

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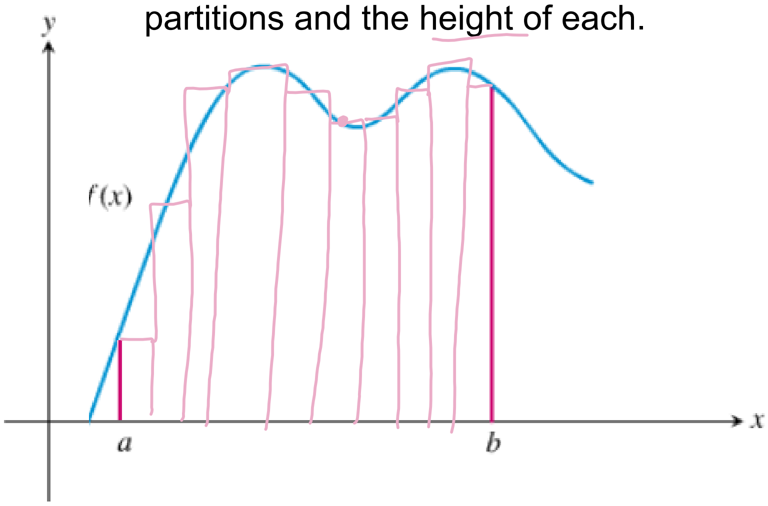
RRAM, LRAM and MRAM are all types of Riemann sums. A Riemann sum approximates the area under a curve. It depends on the number of partitions and the height of each.



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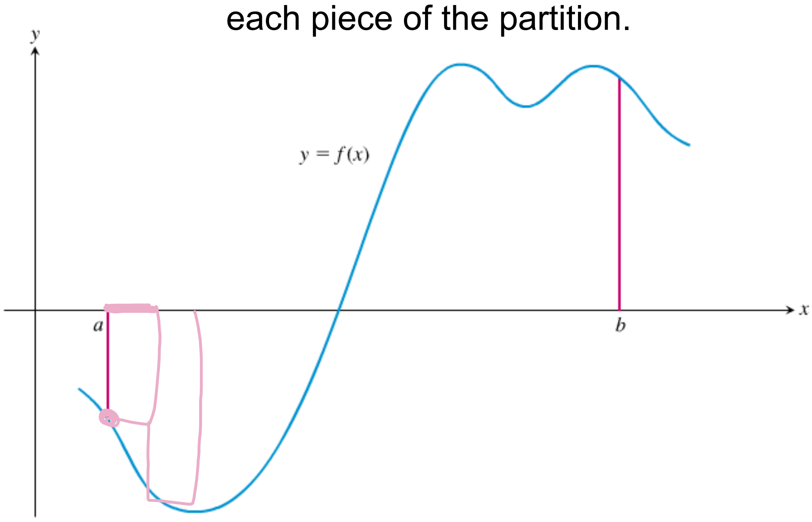
Take a function y = f(x) and pick a continuous interval [a, b]. A Riemann sum is taken by partitioning the interval and taking a rectangular area of each piece of the partition.



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A Riemann sum does not depend on rectangles of equal width.

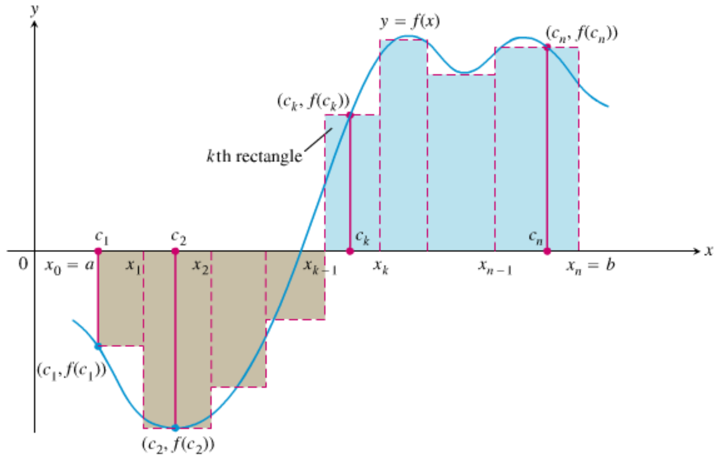

Figure 6.14 Rectangles extending from the x-axis to intersect the curve at the points $(c_k, f(c_k))$. The rectangles approximate the region between the x-axis and the graph of the function.

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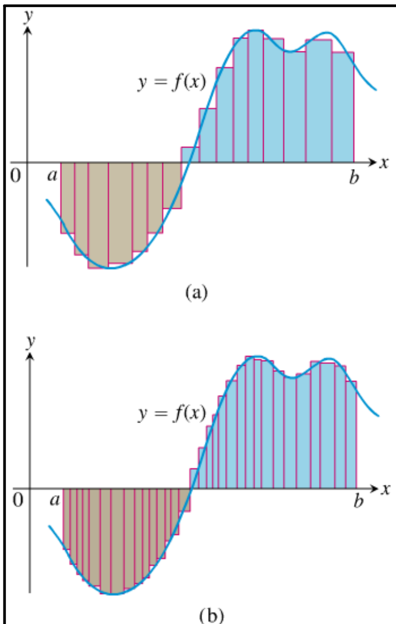

The number of rectangles in the partition can be increased and the heights chosen in such a way that the rectangular area becomes a much more precise estimate for the area under the curve.

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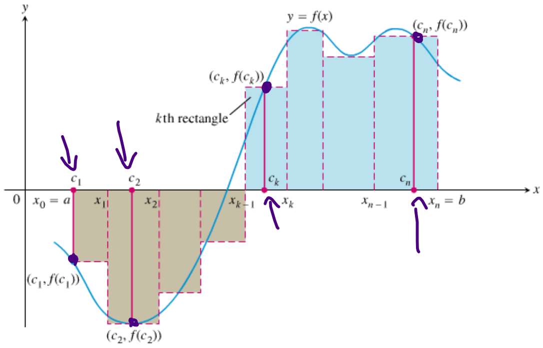
On each subinterval, we form the product $f(c_k) \cdot \Delta x_k$. This product can be positive, negative, or zero, depending on $f(c_k)$.

Finally, we take the sum of these products:

height base

Area $\boxed{S_n} = \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$

This sum, which depends on the partition P and the choice of the numbers c_k, is a **Riemann sum for f on the interval $[a, b]$.**



DEFINITION The Definite Integral as a Limit of Riemann Sums

Let f be a function defined on a closed interval $[a, b]$. For any partition P of $[a, b]$, let the numbers c_k be chosen arbitrarily in the subintervals $[x_{k-1}, x_k]$.

If there exists a number I such that

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k) \Delta x_k = I$$

no matter how P and the c_k's are chosen, then f is **integrable** on $[a, b]$ and I is the **definite integral** of f over $[a, b]$.

THEOREM 1 The Existence of Definite Integrals

All continuous functions are integrable. That is, if a function f is continuous on an interval $[a, b]$, then its definite integral over $[a, b]$ exists.

Approach "actual" area

$$\boxed{\lim_{n\to\infty}\sum_{k=1}^{n} f(c_k)\Delta x} = \int_a^b f(x)\,dx$$

Upper limit of integration

Integral sign

Lower limit of integration

$$\int_a^b f(x)\,dx$$

The function is the **integrand.**

x is the **variable of integration.**

Integral of f from a to b

When you find the value of the integral, you have **evaluated the integral.**

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Express the limits as a definite integral over the given integral

$$\lim_{n\to\infty}\sum_{k=1}^{n} (c_k)^5\,\Delta x \quad [1,2]$$

$$\int_1^2 x^5\,dx$$

$$\lim_{n\to\infty}\sum_{k=1}^{n} c_k+\frac{c_k}{2}\Delta x \quad [-3,5]$$

$$\int_{-3}^{5} x+\frac{x}{2}\,dx$$

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DEFINITION Area Under a Curve (as a Definite Integral)

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ from a to b** is the integral of f from a to b,

$$A = \int_a^b f(x)\,dx.$$

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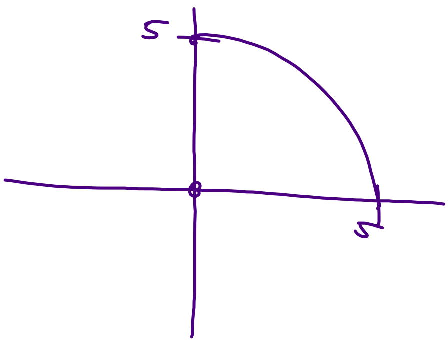
Evaluate the following integral:

$$\int_0^5 \sqrt{25 - x^2}\,dx = \frac{1}{4}\pi 5^2$$



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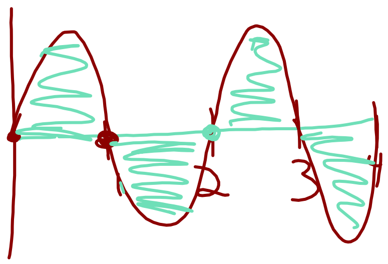
A particle, starting at $x = 12$, travels along the x-axis during the time interval $0 \le t \le 4$ with its velocity given by $v(t) = \sin(\pi t)$. Determine the particle's position on the axis at $t = 4$

$$12 + \int_0^4 \sin(\pi t)\,dt = 12 + 0 = 12$$



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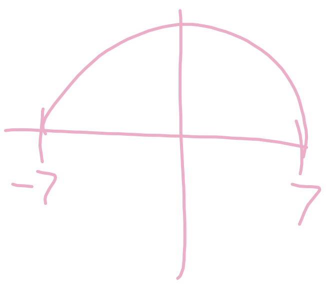
Evaluate the following integral:

$$\int_{-7}^{7} \sqrt{49 - x^2}\,dx$$



$\frac{1}{2}\pi \cdot 7^2$

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Evaluate the following integral:

$$\int_{-7}^{7} -\sqrt{49 - x^2}\, dx$$

$-\frac{1}{2}\pi \cdot 7^2$

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$$\int_a^b f(x)\,dx = (\text{area above the } x\text{-axis}) - (\text{area below the } x\text{-axis}).$$

$$\text{Area} = -\int_a^b f(x)\,dx \quad \text{when} \quad f(x) \le 0.$$

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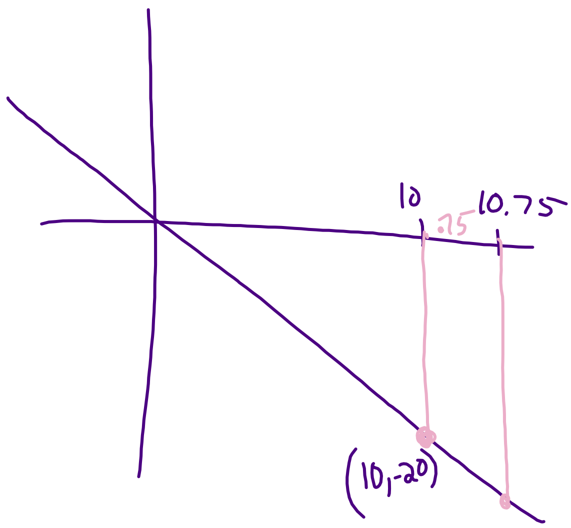
Evaluate the following integral:

$$\int_{10}^{10.75} -2x \, dx$$

$\frac{1}{2}(.75)(-20 - 21.5)$



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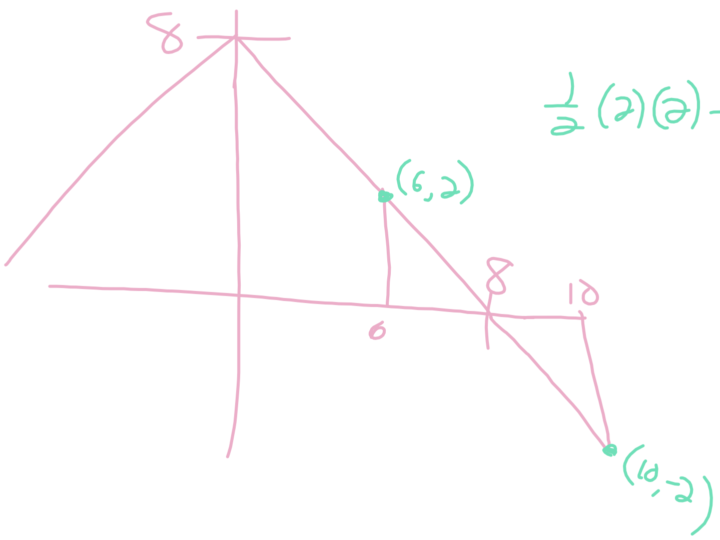
Evaluate the following integral:

$$\int_{6}^{10} 8 - |m| \, dm$$

$\frac{1}{2}(2)(2) - \frac{1}{2}(2)(2) = 0$



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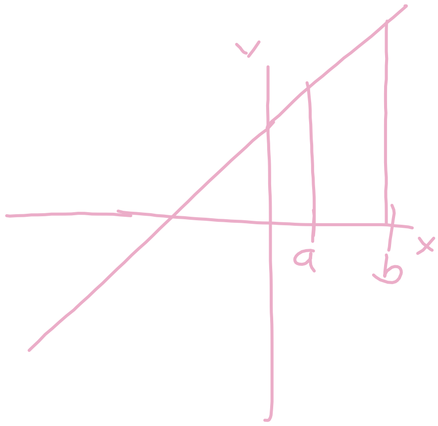
Evaluate the following integral:

$$\int_a^b (q + 5)\, dq \quad , \quad b>a>0$$



$$\frac{1}{2}(b-a)(a+5+b+5)$$

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Evaluate the following integral using NINT (calculator):

$$\int_3^{12} 7x^2 - 1\, dx$$

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Evaluate the following integral using NINT (calculator):

$$\int_{0.57}^{2\sqrt{5}} x^3 - 5x^{-3}\, dx$$

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Homework

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1 - 33 odd, 34, 37- 44 all, 46 - 51 all

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