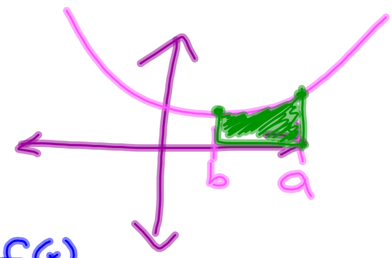
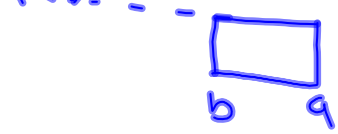
AP Calculus

Chapter 6

Section 6-3

1. *Order of Integration:* $\int_b^a f(x)\,dx =$





$f(x)(a-b)$

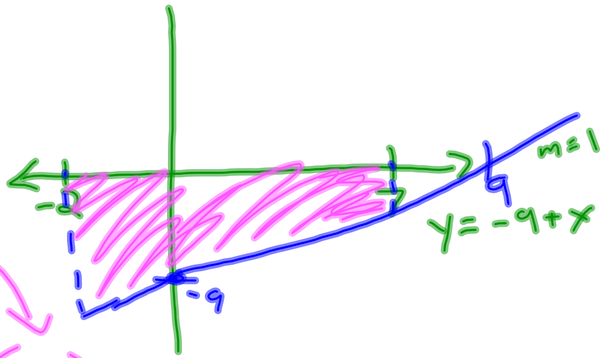
$\int_a^b f(x)dx$ $\quad f(x)(b-a)$

Evaluate the following integral:

$$\int_{7}^{-2} -9 + x \, dx$$

upper (-2)

lower (7)



$(-2-7)$

$\frac{1}{2}(-11+-2)(-9)$

$\frac{1}{2}(-13)(-9)$

$\frac{117}{2}$

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. *Zero:* $\int_a^a f(x)\,dx =$

$f(a) \cdot 0 = 0$

Evaluate the following integral:

$$\int_{10}^{10}\left[-2x^4-13\cos^3(x+2)\right]dx$$

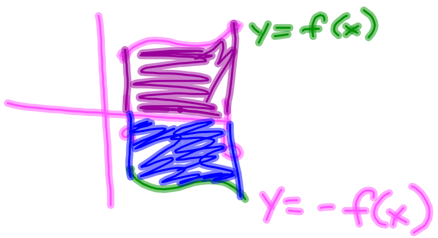
$= 0$

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. *Zero:* $\int_a^a f(x)\,dx = 0$

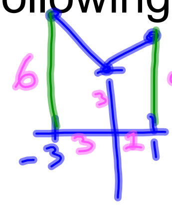
3. *Constant Multiple:* $\int_a^b kf(x)\,dx =$

$\int_a^b -f(x)\,dx =$



Evaluate the following integrals:

$$\int_{-3}^{1} 3 + |k|\, dk =$$



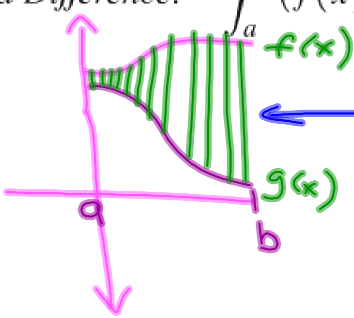
$$\tfrac{1}{2}(6+3)(3)+\tfrac{1}{2}(4+3)(1)$$
$$\tfrac{27}{2}+\tfrac{7}{2}=\tfrac{34}{2}=17$$

$$\int_{-3}^{1} 6 + 2|k|\, dk \qquad 2\int_{-3}^{1} 3+|k|\,dk = 34$$

$$\int_{-3}^{1} -3 - |p|\, dp \qquad -\int_{-3}^{1} 3+|p|\,dp = -17$$

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
2. *Zero:* $\int_a^a f(x)\,dx = 0$
3. *Constant Multiple:* $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

   $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$
4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$



$$\int_a^b f(x)-g(x)\,dx$$

Evaluate the following integral:

$$\int_0^2 \left[7q - |q| + \sqrt{4 - q^2}\right] dq$$

$$\int_0^2 \underline{7q}\,dq - \int_0^2 |q|\,dq + \int_0^2 \sqrt{4-q^2}\,dq$$

$$\cancel{\tfrac{1}{2}}\cancel{(2)}(14) - \cancel{\tfrac{1}{2}}\cancel{(2)}(2) + \frac{\pi \cancel{4}}{\cancel{4}}$$

$$12 + \pi$$

1. *Order of Integration:* $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

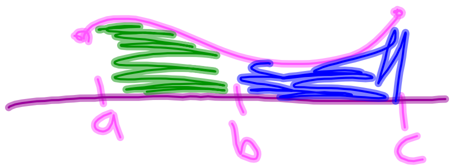
2. *Zero:* $\displaystyle\int_a^a f(x)\,dx = 0$

3. *Constant Multiple:* $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

$\displaystyle\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$

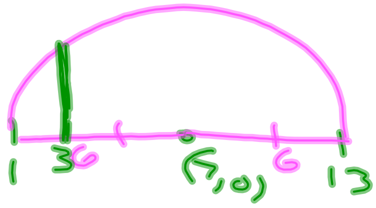
4. *Sum and Difference:* $\displaystyle\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx =$



Evaluate the following integral:

$$\int_1^3 \sqrt{36 - (x-7)^2}\,dx + \int_3^{13} \sqrt{36 - (x-7)^2}\,dx$$



$\frac{1}{2}\pi 36 = 18\pi$

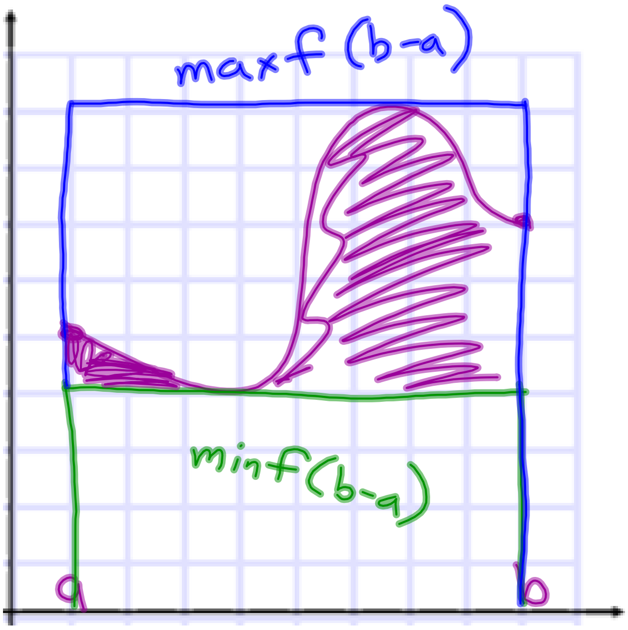
6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b-a) \leq \int_a^b f(x)\,dx \leq \max f \cdot (b-a).$$

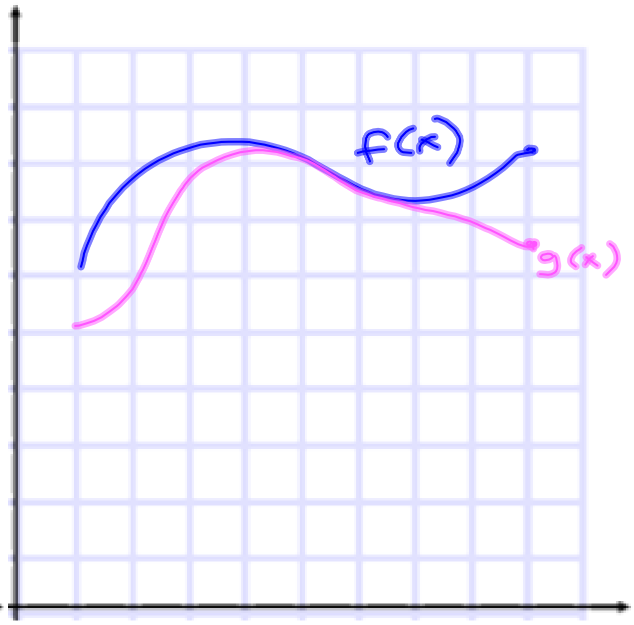
7. *Domination:* $f(x) \geq g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

$$f(x) \geq 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \geq 0$$ $g = 0$

#6



#7



Using the max-min inequality rule, determine an interval that the value of the integral falls within?

$$\int_3^{3\pi} \underbrace{1 - \cos^2(x + \pi)}_{\substack{max: 1 \\ min: 0}}\, dx$$

$0 \le \cos^2 \le 1$

$$0(3\pi - 3) \le \int_3^{3\pi} 1 - \cos^2(x+\pi)\,dx \le 1(3\pi - 3)$$

$$0 \le \int_3^{3\pi} 1 - \cos^2(x+\pi)\,dx \le 3\pi - 3$$

DEFINITION The Definite Integral as a Limit of Riemann Sums

Let f be a function defined on a closed interval $[a, b]$. For any partition P of $[a, b]$, let the numbers c_k be chosen arbitrarily in the subintervals $[x_{k-1}, x_k]$.

If there exists a number I such that

$\|P\|$

$$\lim_{\|P\|\to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k = I$$

no matter how P and the c_k's are chosen, then f is **integrable** on $[a, b]$ and I is the **definite integral** of f over $[a, b]$.

$$\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx$$

- Partition the interval [2,5] into six regions. What is the average length of each region?

$$\frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$$

-

$$\frac{8-(-5)}{100} = \frac{13}{100}$$

-

$$\frac{b-a}{n}$$

$$\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx \qquad \Delta x = \frac{b-a}{n}$$

- Use MRAM to estimate the area under the curve y = 5x⁵ from [-2.5, 1.5] using four rectangles of equal width. What is the average area of each rectangle?

$$\frac{\frac{1.5+2.5}{4}\left(f(-2)+\dots f(1)\right)}{4} = \frac{1(-160+-5+0+5)}{4} = -40$$

-

$$\frac{1(f(6)+f(7)\dots+f(11))}{6}$$

-

$$\frac{1(f(c_1)+f(c_2)+f(c_3)+\dots+f(c_n))}{n}$$

$$\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx$$

$$\frac{1}{b-a}\cdot\Delta x = \frac{\cancel{b-a}}{n}\cdot\frac{1}{\cancel{b-a}}$$

$$\frac{\Delta x}{b-a} = \frac{1}{n}$$

$$\frac{1}{n}\cdot\sum_{k=1}^{n} f(c_k) = \frac{\Delta x}{b-a}\sum_{k=1}^{n} f(c_k)\Delta x$$

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

DEFINITION Average (Mean) Value

If f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

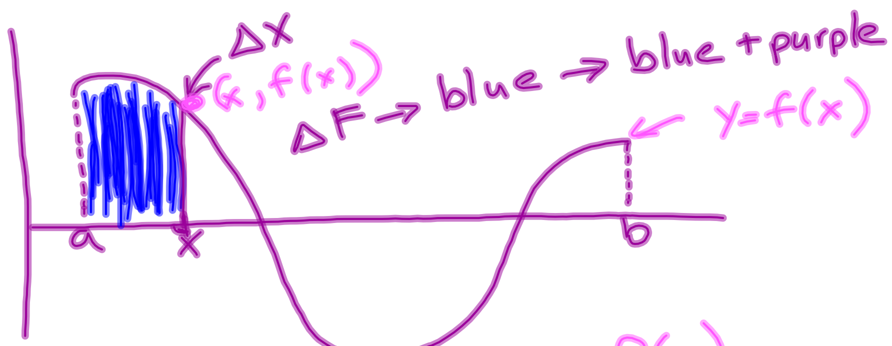
$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

THEOREM 3 The Mean Value Theorem for Definite Integrals

If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Do exploration 2 on page 293



$f(x)$

$$F(x) = \left(\int_a^x f(t)\,dt\right) = f(x)$$

$$F'(x) = f(x)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

This means that the integral is an *antiderivative* of f, a fact we can exploit in the following way.

If F is any antiderivative of f, then

$$\int_a^x f(t)\,dt = F(x) + C$$

$$\int_a^a f(t)\,dt = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a).$$

$$\int_a^x f(t)\,dt = F(x) - F(a).$$

Find the value of each integral

$$\int_0^1 6v^5 - 4v^2\,dv = F(v)$$

$$v^6 - \frac{4}{3}v^3 = F(v)$$

$$F(1) - F(0)$$

$$1^6 - \frac{4}{3}(1)^3 - \left(\cancel{0^6} - \frac{4}{3}\cancel{(0)^3}\right)$$

$$-\frac{1}{3}$$

$$\int_0^{\pi} 1 + 9\sin(k)\,dk$$

$$k - 9\cos k = F(k)$$

$$F(\pi) - F(0)$$

$$\pi - 9\cos\pi - (\cancel{0} - 9\cos 0)$$

$$\pi + 9 + 9$$

$$\pi + 18$$

Homework

Pages 294 - 296

1 - 7 odd, 8, 11 - 37 odd, 45 - 50 all