AP Calculus

Chapter 6

Section 6-4

May 13-10:02 PM

THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

constant

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

THEOREM 3 The Mean Value Theorem for Definite Integrals

If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Mar 3-2:01 PM

DEFINITION Average (Mean) Value

If f is integrable on $[a, b]$, its **average (mean) value** on $[a, b]$ is

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

THEOREM 3 The Mean Value Theorem for Definite Integrals

If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

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Proof of the FToC, P1

$$F(x) = \int_a^x f(t)\,dt$$

$$F'(x) = \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$

$$= \lim_{h\to 0}\frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h}$$

$$\lim_{h\to 0}\frac{1}{h}\int_x^{x+h} f(t)dt$$ ← avg value of $f(t)$

$x \le c \le x+h$

as $h\to 0$: $x\to c$

$$\lim_{h\to 0} f(c) = f(c)$$

Therefore:

$$F'(x) = f(c) = f(x)$$

Mar 4-7:45 AM

Using the FToC

Find...

$$\frac{d}{dx}\int_5^x 3t^{-1}\,dt = 3x^{-1}$$

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Using the FToC with the Chain Rule

Find...

$$F'(x) = \frac{d}{dx}\int_0^{x^3} \sin(t)\,dt \longrightarrow u = x^3$$

$$\frac{dF}{du} = \frac{d}{du}\int_0^u \sin(t)\,dt = \sin(u)$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{dF}{dx} = \frac{dF}{du}\cdot\frac{du}{dx} = \sin(x^3)\cdot 3x^2$$

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Using the FToC with the Variable Lower Limits

Find...

$$\frac{d}{dx}\int_{\sqrt{x}}^{5x} 3t\,e^t\,dt$$

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{a} 3te^t dt + \int_{a}^{5x} 3te^t dt\right)$$

$$\frac{d}{dx}\left(-\int_{a}^{\sqrt{x}} 3te^t dt + \int_{a}^{5x} 3te^t dt\right)$$

$$-3\sqrt{x}e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} + 3(5x)e^{5x} \cdot 5$$

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Constructing a function with Given Derivative and Value

Find a function, y = F(x), whose derivative is sec (x) and satisfies the condition F(6) = 2.

$$F(x) = \int_{6}^{x} \sec(t)\,dt + 2$$

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THEOREM 4 (continued) The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

$$F(x) = \int_a^x f(t)\,dt$$

$$F(b) = \int_a^b f(t)\,dt \qquad F(a) = \int_a^a f(t)\,dt = 0$$

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Evaluate the integral using antiderivatives

$$\int_1^3 4x^3 - x^{-3}\,dx$$

$$x^4 + \frac{1}{2}x^{-2}\Big|_1^3 = 3^4 + \frac{1}{2}(3)^{-2} - \left[1^4 + \frac{1}{2}(1)^{-2}\right]$$

$$81 + \frac{1}{18} - \left(1 + \frac{1}{2}\right) \quad \left(\tfrac{9}{18}\right)$$

$$80 - \tfrac{4}{9} = 79\tfrac{5}{9}$$

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Evaluate the integral

$$\int_{\pi/4}^{3\pi/4} \frac{1}{(x + \pi/4)^2} - \csc^2(x)\, dx$$

$$\int_{\pi/4}^{3\pi/4} (x+\pi/4)^{-2} - \csc^2(x)\, dx$$

$$-(x+\pi/4)^{-1} + \cot(x) \Big|_{\pi/4}^{3\pi/4}$$

$$-(\pi)^{-1} + \cot(3\pi/4) - \left[-(\pi/2)^{-1} + \cot(\pi/4)\right]$$

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Writing Assignment: Using Antiderivatives

EXAMPLE: $\int \cos u\, du = \sin u + C$

- Write 15 formulas like the one given above based on our rules of differentiation.
- Each should be indefinite (no boundary indicated)
- Each formula should account for the constant
- Writing: The function y = tan (x) is the derivative of some function g(x) (by the FToC). Explain completely what this means for the domain of g(x).

Due Monday 3/6

Mar 4-8:12 AM

Homework

Pages 306 - 308

1 - 39 odd, 45 - 50 all, 58

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