AP Calculus

Chapter 7

Section 7-1

DEFINITION Differential Equation

An equation involving a derivative is called a **differential equation.** The **order of a differential equation** is the order of the highest derivative involved in the equation.

EX:

- $\frac{dy}{dx} = x - 5$
- $\frac{d^2y}{dx^2} = 2y - 3x$

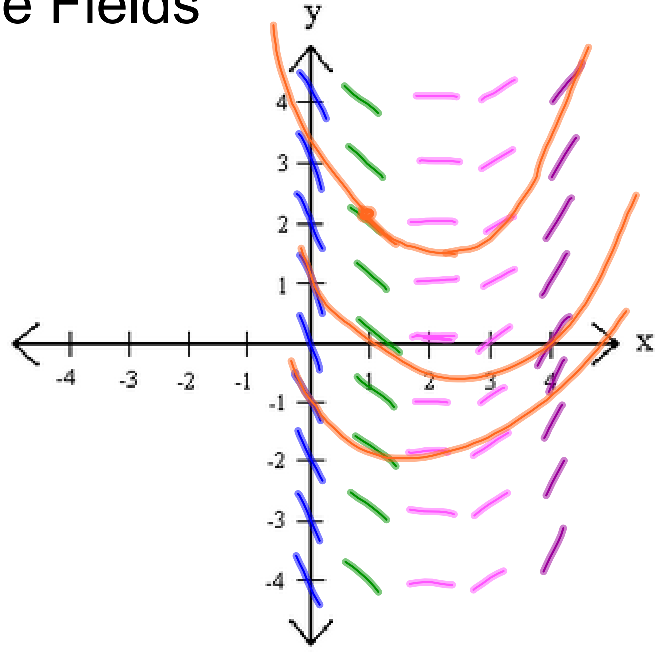
Slope Fields

$$\frac{dy}{dx} = x - 2$$

$y' \to f'(x)$

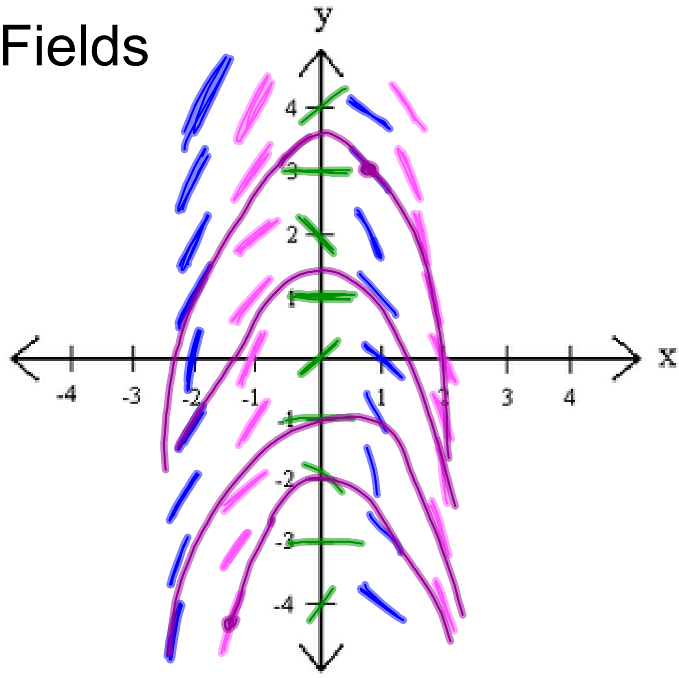
$$\int 1 \frac{dy}{dx} = \int x - 2 \cdot dx$$

$$y = \frac{1}{2}x^2 - 2x + c$$



Slope Fields

$$\frac{dy}{dx} = \cos\left(\frac{\pi}{2}y\right) - 2x$$



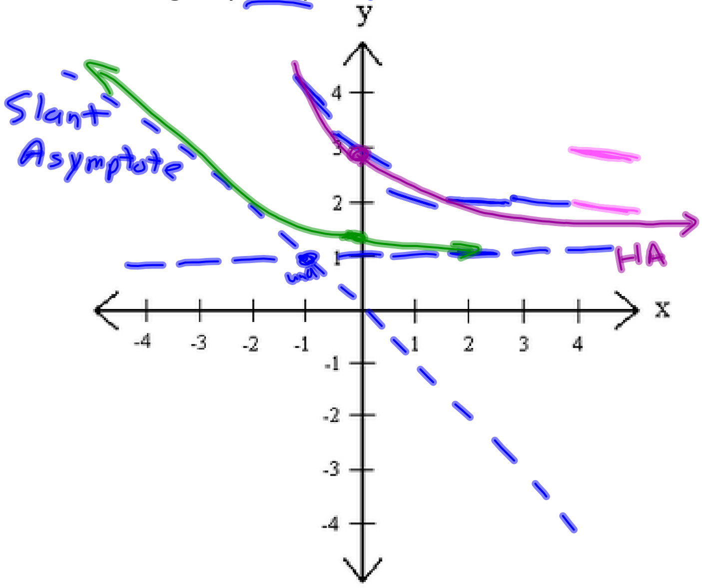
Sketch the graph of a particular solution to the differential equation that passes through (0, 3) ← point

$$\frac{dy}{dx} = \frac{1 - y}{x + y}$$

undefined

$x + y = 0$

$y = -x$



EXPLORATION 1 Seeing the Slopes **Page 327**

Figure 7.1 shows the general solution to the exact differential equation $dy/dx = \cos x$.

1. Since $\cos x = 0$ at odd multiples of $\pi/2$, we should "see" that $dy/dx = 0$ at the odd multiples of $\pi/2$ in Figure 7.1. Is that true? How can you tell?
2. Algebraically, the y-coordinate does not affect the value of $dy/dx = \cos x$. Why not?
3. Does the graph show that the y-coordinate does not affect the value of dy/dx? How can you tell?
4. According to the differential equation $dy/dx = \cos x$, what should be the slope of the solution curves when $x = 0$? Can you see this in the graph?
5. According to the differential equation $dy/dx = \cos x$, what should be the slope of the solution curves when $x = \pi$? Can you see this in the graph?
6. Since $\cos x$ is an even function, the slope at any point should be the same as the slope at its reflection across the y-axis. Is this true? How can you tell?

Initial Value Problems

Find all functions that satisfy the given differential equation. Then find the particular solution to the equation whose graph passes through (3, -4).

$$\frac{dy}{dx} = 6x^2 - 17$$

Initial Value Problems

Find all functions that satisfy the given differential equation. Then find the particular solution to the equation whose graph passes through (1, 2).

$$\frac{dy}{dx} = \frac{3}{x} - e^{x-1}$$

Euler's Method for Graphing a Solution to an Initial Value Problem

1. Begin at the point (x, y) specified by the initial condition. This point will be on the graph, as required.
2. Use the differential equation to find the slope dy/dx at the point.
3. Increase x by a small amount Δx. Increase y by a small amount Δy, where $\Delta y = (dy/dx)\Delta x$. This defines a new point $(x + \Delta x, y + \Delta y)$ that lies along the linearization (Figure 7.6).
4. Using this new point, return to step 2. Repeating the process constructs the graph to the right of the initial point.
5. To construct the graph moving to the left from the initial point, repeat the process using negative values for Δx.

Let f be the function that satisfies the initial value problem in Example 6 (that is, $dy/dx = x + y$ and $f(2) = 0$). Use Euler's Method and increments of $\Delta x = 0.2$ to approximate $f(3)$.

(x, y)	$dy/dx = x + y$	Δx	$\Delta y = (dy/dx)\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 0)	2	0.2	0.4	(2.2, 0.4)
(2.2, 0.4)	2.6	0.2	0.52	(2.4, 0.92)
(2.4, 0.92)	3.32	0.2	0.664	(2.6, 1.584)
(2.6, 1.584)	4.184	0.2	0.8368	(2.8, 2.4208)
(2.8, 2.4208)	5.2208	0.2	1.04416	(3, 3.46496)

A particle moving across the line $y = 5$ has an initial coordinate of (2, 5). The rate that the particle's position changes is given by the equation:

$$\frac{ds}{dt} = st + 1$$

Use Euler's Method with increments of $\Delta t = 0.25$ to approximate the particle's position at time $t = 2$.

Homework

Pages 331 - 334

1 - 10 all, 11 - 21 odd, 25 - 28 all, 29 - 45 odd, 51, 53, 69 - 74 all