# AP Calculus

Chapter 7

Section 7-2

**Properties of Indefinite Integrals**

$$\int k\, f(x)dx = k\int f(x)\, dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Evaluating an indefinite integral

$$\int 3x^2 - 2\sin x \, dx$$

$x^3 + 2\cos x + C$

Evaluating an indefinite integral

$$\int 11\cos x \, dx$$

$11\sin x + C$

**Power Formulas**

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1 \qquad \int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u| + C$$

For $u > 0$, we have $\frac{d}{du}(\ln|u| + C) = \frac{d}{du}(\ln u + C) = \frac{1}{u} + 0 = \frac{1}{u}$.

For $u < 0$, we have $\frac{d}{du}(\ln|u| + C) = \frac{d}{du}(\ln(-u) + C) = \frac{1}{-u}(-1) + 0 = \frac{1}{u}$.

# Evaluating an indefinite integral

$$\int m^4\,dm$$

$\frac{1}{5}m^5 + c$ or $\frac{m^5}{5} + C$

# Evaluating an indefinite integral

$$\int v^{-2} - v^{-1} + 3\, dv$$

$$-v^{-1} - \ln|v| + 3v + c$$

**Trigonometric Formulas**

$$\int \cos u\, du = \sin u + C \qquad \int \sin u\, du = -\cos u + C$$

$$\int \sec^2 u\, du = \tan u + C \qquad \int \csc^2 u\, du = -\cot u + C$$

$$\int \sec u \tan u\, du = \sec u + C \qquad \int \csc u \cot u\, du = -\csc u + C$$

## Evaluating an indefinite integral

$$\int \cos u + 10u \, du$$

$$-\sin u + 5u^2 + C$$

## Evaluating an indefinite integral

$$\int (\sec x + \tan x)\sec x \, dx$$

$$\int \sec^2 x + \sec x \tan x \, dx$$

$$\tan x + \sec x + C$$

**Exponential and Logarithmic Formulas**

$$\int e^u\,du = e^u + C \qquad \int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \ln u\,du = u\ln u - u + C \quad \text{(See Example 2)}$$

$$\int \log_a u\,du = \int \frac{\ln u}{\ln a}\,du = \frac{u\ln u - u}{\ln a} + C$$

$$\frac{d}{du}(u\ln u - u + C) = 1\cdot\ln u + u\left(\frac{1}{u}\right) - 1 + 0 = \ln u + 1 - 1 = \ln u.$$

Evaluating an indefinite integral

$$\ln 4 \cdot \int 4^x + \log_4 x\,dx$$

$$\ln 4\left(\frac{4^x}{\ln 4} + \frac{x\ln x - x}{\ln 4} + C\right)$$

$$4^x + x\ln x - x + C$$

# Evaluating an indefinite integral

$$\int 2x^3 + 5^x \, dx$$

$$2\frac{x^4}{4} + \frac{5^x}{\ln 5} + c$$

$$\frac{x^4}{2} + \frac{5^x}{\ln 5} + c$$

# Integrating with respect to correct variable

Let $f(x) = 30x^2 - 4x$ and $v = x^3$. Evaluate the following integrals.

$$\int f(x)\,dx$$

$$10x^3 - 2x^2 + c$$

$$\int f(v)\,dv$$

$$10v^3 - 2v^2 + c$$

$$10x^9 - 2x^6 + c$$

$$\int f(v)\,dx$$

$$\int 30v^2 - 4v\,dx$$

$$\int 30x^6 - 4x^3\,dx$$

$$\frac{30}{7}x^7 - x^4 + c$$

## Evaluating an indefinite integral by substitution

$$\int e^{3x}\,dx$$

$u = 3x$

$du = 3\,dx$

$$\frac{1}{3}\int e^{3x} \cdot 3\,dx$$

$$\frac{1}{3}\int e^{u}\,du$$

$$\frac{1}{3}(e^{u} + c)$$

$$\frac{1}{3}(e^{3x} + c)$$

$$\frac{1}{3}e^{3x} + C$$

## Evaluating an indefinite integral by substitution

$$\frac{1}{2}\int (7x - 2)\cdot 2\sin(7x^2 - 4x + 1)\,dx$$

$u = 7x^2 - 4x + 1$

$du = 14x - 4\,dx$

$du = 2(7x - 2)\,dx$

$$\frac{1}{2}\int \sin(u)\,du$$

$$\frac{1}{2}(-\cos u + c)$$

$$-\frac{1}{2}\cos u + C$$

$$-\frac{1}{2}\cos(7x^2 - 4x + 1) + C$$

# Evaluating a definite integral by substitution

$$-\int_0^{\ln 2} \frac{-e^x}{9 - e^x}\,dx$$

$u = 9 - e^x$

$du = -e^x\,dx$

$u = 9 - e^0 = 8$

$u = 9 - e^{\ln 2} = 7$

$$-\int_8^7 \frac{1}{u}\,du$$

$$\int_7^8 \frac{1}{u}\,du$$

$$\ln u \Big|_7^8 = \ln 8 - \ln 7$$

$$= \ln \frac{8}{7}$$

# Evaluating an indefinite integral by substitution

$$\frac{1}{2}\int \csc^2(2x + 1)\,2\,dx$$

$u = 2x + 1$

$du = 2\,dx$

$$\frac{1}{2}\int \csc^2(u)\,du$$

$$\frac{1}{2}(-\cot(u) + c)$$

$$-\frac{1}{2}\cot(2x+1) + c$$

## Evaluating a definite integral by trig substitution

$$\int_{\pi/6}^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$$

$\frac{\sin x}{\cos x} = \tan x$

$\sec x = \frac{1}{\cos x}$

$\int_{\pi/6}^{\pi/3} \sec x \tan x \, dx$

$\sec x \Big|_{\pi/6}^{\pi/3} = \sec \pi/3 - \sec \pi/6$

## Evaluating a definite integral by trig substitution

$(\cos x)^2$

$$\int_{0}^{\pi} \frac{\sin x}{\sec^2 x}\,dx$$

$- \int_0^\pi \cos^2 x (\sin x)\,dx$

$- \int_1^{-1} u^2 \, du$

$u = \cos x$

$du = -\sin x \, dx$

$\int_{-1}^{1} u^2 \, du$

$\frac{1}{3}u^3 \Big|_{-1}^{1} = \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3}$

## Evaluating an indefinite integral by substitution

$\sin^2 x+\cos^2 x=1$

$$\int \sin^3 x\, dx$$

$$\int \sin^2 x \cdot \sin x\, dx$$

$$\int (1-\cos^2 x) \sin x\, dx$$

$$\int (\cos^2 x-1)(-\sin x)dx$$

$u=\cos x$

$du=-\sin x\, dx$

$$\int u^2-1 \quad du$$

$$\frac{1}{3}u^3-u+C$$

$$\frac{1}{3}\cos^3 x-\cos x+C$$

# Homework

Pages 342 - 343

# 1 - 6 all, 17 - 39 odd, 48 - 60 by 3, 72, 73