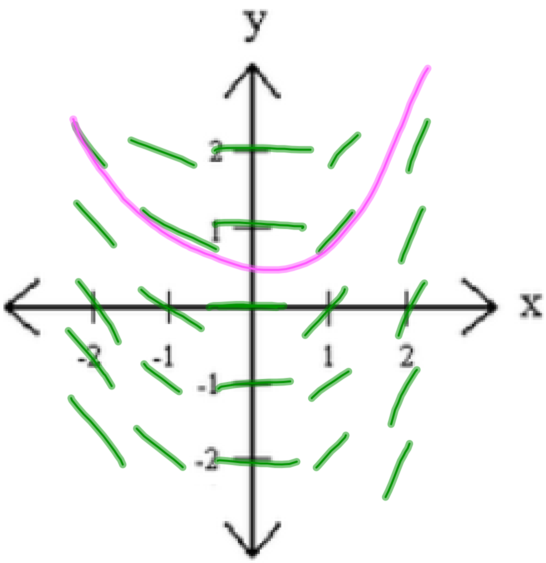
AP Calculus

Chapter 7

Section 7-4

Matching a slope field with an antiderivative.

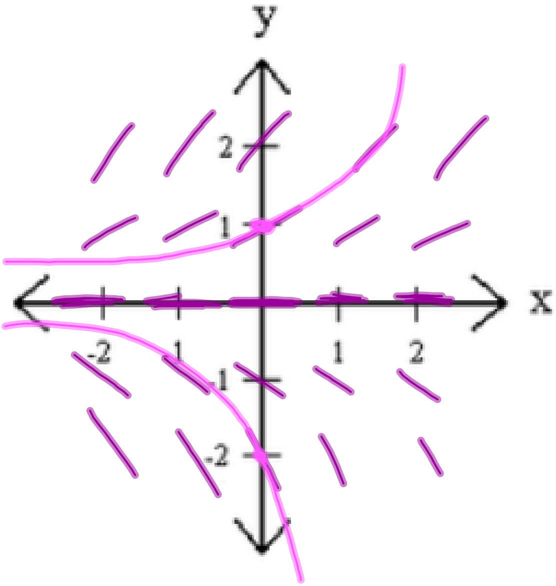


$$\frac{dy}{dx} = 2x$$

$\int dy = \int 2x\,dx$

$y = x^2 + C$

Matching a slope field with an antiderivative.



$$\frac{dy}{dx} = y$$

$$\int \frac{1}{y}\,dy = \int dx$$

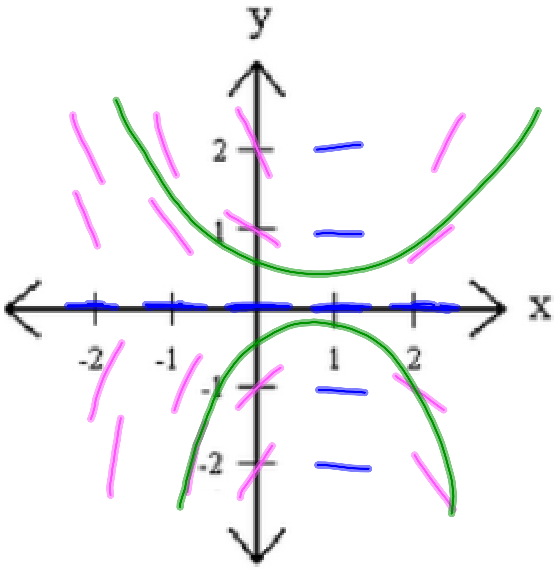
$$\ln|y| = x + c$$

$$|y| = e^{x+c}$$

$$|y| = e^{x} \cdot e^{c}$$

$$y = ae^{x}$$

Matching a slope field with an antiderivative.



$$\frac{dy}{dx} = xy - y$$

$$\frac{dy}{dx} = (x-1)y$$

$$\int \frac{1}{y}\,dy = \int x-1\,dx$$

$$\ln|y| = \frac{1}{2}x^2 - x + c$$

$$y = e^{\frac{1}{2}x^2 - x + c}$$

$$y = ae^{\frac{1}{2}x^2 - x}$$

Solve for y if y = 4 when x = 1.

$$y\frac{dy}{\cancel{dx}} = \frac{1}{x\cancel{y}}\,dx$$

$$\int y\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}y^2 = \ln|x| + c$$

$$y = \sqrt{2\ln|x| + c}$$

$$4 = \sqrt{2\ln 1 + c}$$

$$4 = \sqrt{c}$$

$$16 = c$$

$$y = \sqrt{2\ln|x| + 16}$$

Solve for y if y = $\frac{\pi}{4}$ when x = 0.

$$\frac{dy}{dx} = \tan y$$

$$\int \cot y\,dy = \int dx$$

$$\int \frac{\cos y}{\sin y}\,dy = \int dx$$

$u = \sin y$

$du = \cos y\,dy$

$$\int \frac{du}{u} = \int dx$$

$$\ln|u| = x + c$$

$$\ln|\sin y| = x + c$$

$$\sin y = e^{x+c}$$

$$\sin y = ae^x$$

$y = \sin^{-1}(ae^x)$

$$\sin\frac{\pi}{4} = ae^0$$

$$\frac{\sqrt{2}}{2} = a$$

$$y = \sin^{-1}\left(\frac{\sqrt{2}}{2}e^x\right)$$

Solve the differential equation.

$$\frac{dt}{y}\cdot\frac{dy}{\cancel{dt}} = k\cancel{y}\,dt$$

$$\int \frac{dy}{y} = k\int dt$$

$$e^{\ln|y|} = e^{kt+c}$$

$$y = e^{kt+c} = e^{kt}\cdot e^{c}$$

$$y = ae^{kt}$$

a - initial value

e^{k} - growth/decay factor

The Law of Exponential Change

If y changes at a rate proportional to the amount present (that is, if $dy/dt = ky$), and if $y = y_0$ when $t = 0$, then

$$y = y_0\,e^{kt}.$$

The constant k is the **growth constant** if $k > 0$ or the **decay constant** if $k < 0$.

$$e^{k} > 1$$

$$k > \ln 1 = 0$$

Application: Finding Half-Life $t=?$

Set up and equation and find a general expression for half-life.

$$\frac{\cancel{y_0} e^{-kt}}{\cancel{y_0}} = \frac{\frac{1}{2}\cancel{y_0}}{\cancel{y_0}}$$

$$\frac{-kt}{-k} = \frac{\ln \frac{1}{2}}{-k}$$

$$t = \frac{\ln \frac{1}{2}}{-k}$$

Homework

Pages 361 - 364

1 - 14 all, 27, 28, 46, 47, 48