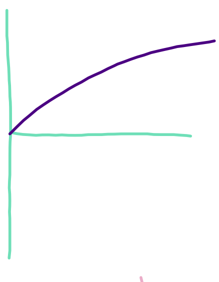
Calculus for life Sciences

Section 5.1

May 13-10:02 PM

* Increasing functions:

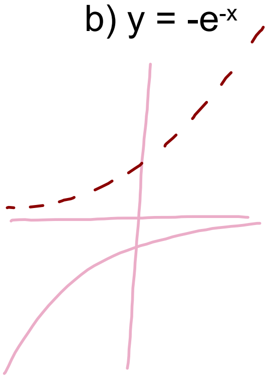
a) $y = \sqrt{x} = x^{1/2}$



$y' = \frac{1}{2}x^{-1/2}$

$y' = \frac{1}{2\sqrt{x}}$

b) $y = -e^{-x}$

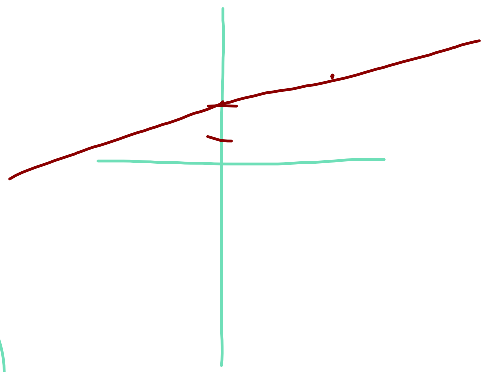


$y' = \cancel{-}e^{-x}(\cancel{-}1)$

$y = e^{-x}$

$y = \frac{1}{e^{x}}$

c) $y = 0.25x+2$



$y' = .25$

$y' = \frac{1}{4}$

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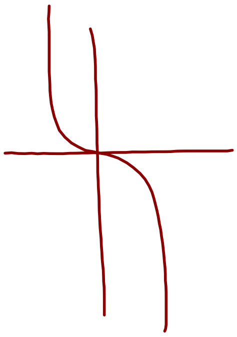
Decreasing functions:

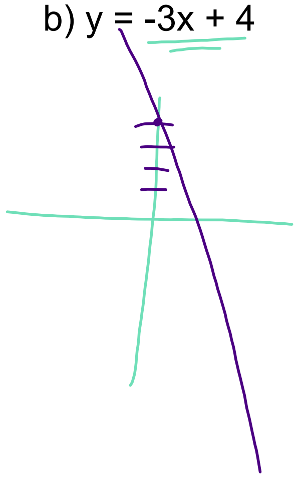
a) $y = -x^3 = (-x)^3$



$-3x^2$

$(-\infty, \infty)$

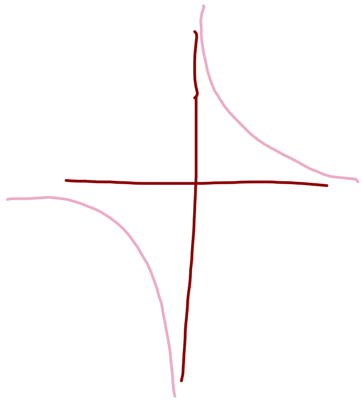
b) $y = -3x + 4$



-3

$(-\infty, \infty)$

c) $y = x^{-1} = \frac{1}{x}$



$-x^{-2} = -\frac{1}{x^2}$

$(-\infty, 0) \cup (0, \infty)$

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Test for Intervals

Where f(x) is increasing and decreasing:

$f(x) = 2x^2 + 1$

Suppose a function *f* has a derivative at each point in an open interval; then:

Der orig

- if $f'(x) > 0$ for each x in the interval, *f* is increasing on the interval
- if $f'(x) < 0$ for each x in the interval, *f* is decreasing on the interval
- if $f'(x) = 0$ for each x in the interval, *f* is constant on the interval.

$f'(x) = 4x$

$0 = 4x$

$x = 0$

Inc: $[0, \infty)$

Dec: $(-\infty, 0]$

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Critical Values:

The *critical numbers* for a function *f* are those numbers, such as c, in the domain of *f* for which $f'(c) = 0$ or $f'(c)$ does not exist.

A *critical point* is a point whose x-coordinate is a critical number, c, and whose y-coordinate is *f*(c).

Applying the Test Using Critical Numbers:

1) Locate the critical numbers for *f* on a number line, as well as any points where *f* is undefined. These points determine intervals to test.

2a) Choose a value of x in each of the intervals determined in step 1. Use these values to determine whether $f'(x) > 0$ or < 0 in that interval. Use the increasing and decreasing test for intervals from last slide.

OR...

2a) Determine analytically (with polynomial end behavior) when $f'(x) > 0$ or < 0.

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Determine when each function is increasing/decreasing.

a) $y = x^4 - 5$

$y' = 4x^3$

$4x^3 = 0$

$x = 0$

− +
-1 0 1

D: $(-\infty, 0]$

I: $[0, \infty)$

b) $y = x^3 - 12x^2 + 45x - 11$

$y' = 3x^2 - 24x + 45$

$y' = 3(x^2 - 8x + 15)$

$y' = \cancel{3}(x-5)(x-3)$

$\cancel{3 \neq 0}$ $x - 5 = 0$ $x - 3 = 0$

$x = 5, 3$

+ − +
0 3 4 5 6

I: $(-\infty, 3] \cup [5, \infty)$

D: $[3, 5]$

c) $y = x + 6.75x^4$

$y' = 1 + 27x^3$

$0 = 1 + 27x^3$

$-1 = 27x^3$

$\sqrt[3]{-\frac{1}{27}} = \sqrt[3]{x^3}$

$-\frac{1}{3} = x$

− +
$-1/3$

D: $(-\infty, -1/3]$

I: $[-1/3, \infty)$

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Find critical numbers and determine when f(x) is increasing and decreasing if $f(x) = (x-1)^{2/3}$ $\rightarrow \sqrt[3]{(x-1)^2}$

$$f'(x) = \frac{2}{3}(x-1)^{-1/3}(1)$$

$$0 = \frac{2}{3(x-1)^{1/3}}$$

$x \neq 1$

CN $x=1$

$-$ | $+$ (test points 0, 2; split at 1)

D: $(-\infty, 1]$

I: $[1, \infty)$

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Determine when each function is increasing/decreasing.

a) $y = x^5 - 5x^3 - 20x$

$5x^4 - 15x^2 - 20$

$5(x^4 - 3x^2 - 4)$

$(x^2-4)(x^2+1)$

$x^2 - 4 = 0$

$x^2 = 4$

$x = 2, -2$

$x^2 = -1$ DNE

$+$ | $-$ | $+$ (test points -3, 0, 3; split at -2, 2)

I: $(-\infty, -3] \cup [2, \infty)$

D: $[-2, 2]$

b) $y = 75x^3 - 2x$

$225x^2 - 2 = 0$

$\cancel{225}x^2 = 2$

$x^2 = 225$

$x = \sqrt{\frac{2}{225}}$

$x = \frac{\sqrt{2}}{15}$

$+$ | $-$ | $+$ (split at $-\frac{\sqrt{2}}{15}$, $\frac{\sqrt{2}}{15}$)

I: $(-\infty, -\frac{\sqrt{2}}{15}] \cup [\frac{\sqrt{2}}{15}, \infty)$

D: $[-\frac{\sqrt{2}}{15}, \frac{\sqrt{2}}{15}]$

c) $y = \ln(x) + x^{-1} + x^{-2}$

$\frac{1}{x} - 1x^{-2} - 2x^{-3}$

$x^3\left(\frac{1}{x} - \frac{1}{x^2} - \frac{2}{x^3}\right) = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1)$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$

$-$ | $+$ | $-$ | $+$ (split at -1, 0, 2)

I: $(-1, 0] \cup [2, \infty)$

D: $(-\infty, -1] \cup [0, 2)$

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Determine when each function is increasing/decreasing.

a) $\cot\left(\frac{1}{2^x - x\ln 2}\right)$ b) $y = e^{x^3 - 12x}$ c) $\sqrt[5]{x - 4}\ \sqrt[3]{2x}$

a)

$$-\csc^2\left(\frac{1}{2^x - x\ln 2}\right) \cdot \left(\frac{-(2^x \cdot \ln 2 - \ln 2)}{(2^x - x\ln 2)^2}\right)$$

$$-2^x \ln 2 + \ln 2 = 0$$

$$\ln 2(-2^x + 1) = 0$$

$$-2^x + 1 = 0$$

$$2^0 = 1 = 2^x$$

$$x = 0$$

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Determine when each function is increasing/decreasing.

a) $\cot\left(\frac{1}{2^x - x\ln 2}\right)$ b) $y = e^{x^3 - 12x}$ c) $\sqrt[5]{x - 4}\ \sqrt[3]{2x}$

b) $y' = e^{x^3 - 12x}(3x^2 - 12)$

$$3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

+ − +

−3 −2 0 2 3

Inc: $(-\infty, -2] \cup [2, \infty)$

Dec: $[-2, 2]$

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Determine when each function is increasing/decreasing.

a) $\cot\left(\frac{1}{2^x - x\ln 2}\right)$ b) $y = e^{x^3 - 12x}$ c) $\sqrt[5]{x-4}\,\sqrt[3]{2x}$

c) $(x-4)^{1/5} \cdot (2x)^{1/3}$

$\frac{1}{5}(x-4)^{-4/5}(2x)^{1/3} + (x-4)^{1/5} \cdot \frac{1}{3}(2x)^{-2/3} \cdot 2$

$\frac{(2x)^{1/3}}{5(x-4)^{4/5}} + \frac{2(x-4)^{1/5}}{3(2x)^{2/3}} = 0$

$(16x-40)$

$\frac{3(2x)+10(x-4)}{15(x-4)^{4/5}(2x)^{2/3}}$ → $6x+10x-40=0$

$16x=40$

$x=2.5$

CN: $x=4, x=0$

− − + +

−1 0 1 2.5 3 4 5

D: $(-\infty, 2.5]$

I: $[2.5, \infty)$

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Complete exercise #54 on page 271.

$K(t) = \frac{4t}{3t^2+27}$

$\frac{(3t^2+27)(4) - 4t(6t)}{(3t^2+27)^2}$

$\frac{12t^2+108-24t^2}{(3t^2+27)^2}$

$\frac{-12t^2+108}{(3t^2+27)^2} = \frac{-12(t^2-9)}{(3t^2+27)^2}$

$t^2-9=0 \quad t^2=9 \quad t=\pm 3$

\+ −

0 1 3 4

$I=[0,3]$

$D=[3,\infty)$

Feb 21-2:04 PM

Homework

Pages 270 - 271

1 - 39 odd, 42 - 45 all, 55

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