Geometry

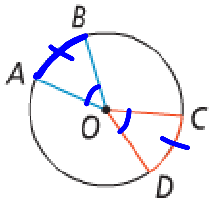
Chapter 12

Section 12-2

Theorem 12-4 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent central angles.



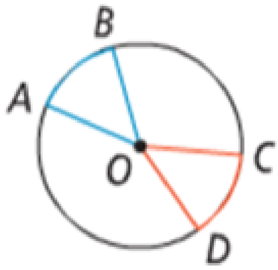
If $\angle AOB \cong \angle COD$, then $\widehat{AB} \cong \widehat{CD}$.

If $\widehat{AB} \cong \widehat{CD}$, then $\angle AOB \cong \angle COD$.

$$L: \frac{m\,Arc}{360} 2\pi r$$

Proof:

Statements	Reasons
$\angle AOB \cong \angle COD$	Given
$\frac{m\angle AOB}{360} = \frac{m\angle COD}{360}$	Div. prop
$\frac{m\overset{\frown}{AB}}{360} = \frac{m\overset{\frown}{CD}}{360}$	sub
$\overline{AO} \cong \overline{CO}$	def radius
$\frac{m\overline{AB}}{360} 2\pi \overline{AO} = \frac{m\overset{\frown}{CD}}{360} 2\pi \overline{CO}$	mult. prop
$\overset{\frown}{AB} \cong \overset{\frown}{CD}$	def. of $\cong$

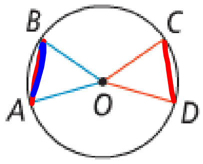


If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

Theorem 12-5

Theorem
Within a circle or in congruent circles, congruent central angles have congruent chords.

Converse
Within a circle or in congruent circles, congruent chords have congruent central angles.



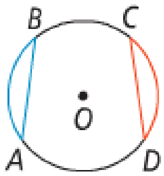
If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

Theorem 12-6

Theorem
Within a circle or in congruent circles, congruent chords have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent chords.

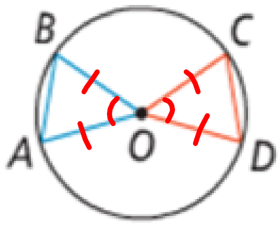


If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.

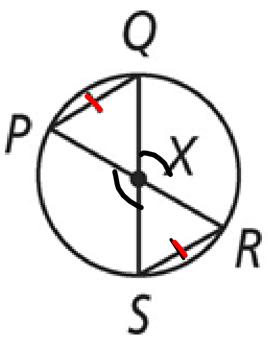
Proof:

Statements	Reasons
$\angle AOB \cong \angle COD$	Given
$\overline{AO} \cong \overline{BO} \cong \overline{CO} \cong \overline{DO}$	def. radius
$\triangle ABO \cong \triangle CDO$	SAS
$\overline{AB} \cong \overline{CD}$	Corr parts $\cong \triangle$



If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.

What conclusion can you draw from each diagram?



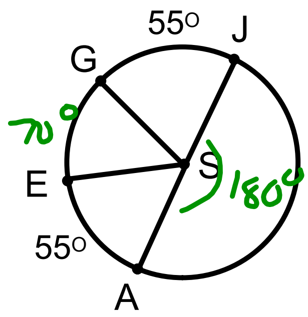
$\angle PXQ \cong \angle SXR$

$\angle QXR \cong \angle PXS$ → vert $\angle s$

$\overset{\frown}{PQ} \cong \overset{\frown}{SR}$

$\overset{\frown}{SQ} \cong \overset{\frown}{PR}$

$\overline{SX} \cong \overline{RX} \cong \overline{PX} \cong \overline{QX}$



$\angle ESA \cong \angle JSG$

$\overset{\frown}{AE} \cong \overset{\frown}{GJ}$

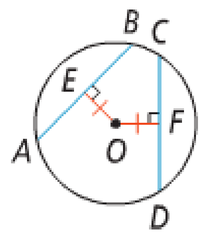
$\overset{\frown}{EG} \cong \overset{\frown}{EG}$

$\overline{ES} \cong \overline{GS} \cong \overline{AS} \cong \overline{JS}$

Theorem 12-7

Theorem
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

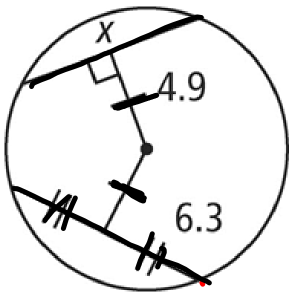
Converse
Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).



If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.

What is the value of x?



$$\begin{array}{r} 6.3 \\ \times\ 2 \\ \hline 12.6 \end{array}$$

$x = 12.6$

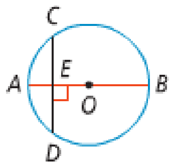
Theorem 12-8

Theorem

In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

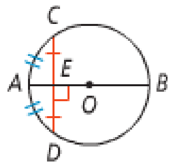
If . . .

$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$



Then . . .

$\overline{CE} \cong \overline{ED}$ and $\overset{\frown}{CA} \cong \overset{\frown}{AD}$



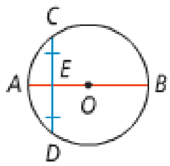
Theorem 12-9

Theorem

In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

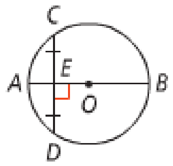
If . . .

$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$



Then . . .

$\overline{AB} \perp \overline{CD}$



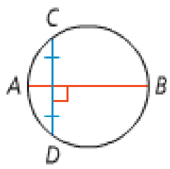
Theorem 12-10

Theorem

In a circle, the perpendicular bisector of a chord contains the center of the circle.

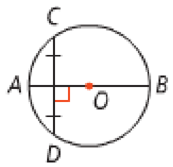
If . . .

$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$

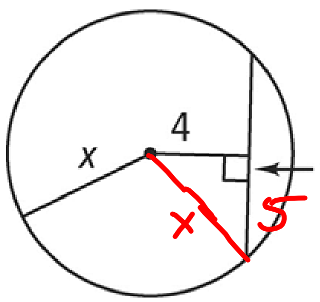


Then . . .

$\overline{AB}$ contains the center of $\odot O$



What is the value of x?

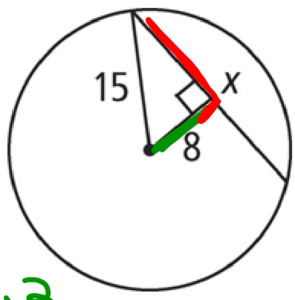


$$4^2+5^2=x^2$$

$$16+25=x^2$$

$$\sqrt{41}=\sqrt{x^2}$$

$$x=6.4$$



$$64+\left(\frac{x}{2}\right)^2=225$$

$$-64 \qquad\qquad -64$$

$$\sqrt{\left(\frac{x}{2}\right)^2}=\sqrt{161}$$

$$2\cdot\frac{x}{2}=12.7\cdot 2$$

$$x=25.4$$

Homework

Pages 776 - 777

7-11 all, 14, 21-24 all